University of Alberta

Optimization Based Forest Planning Tools for Sustainable Forest Management

by

David Mateiyenu Nanang ©

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfilment
of the requirements for the degree of Doctor of Philosophy

in

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329L Michener Park
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T6H 4M5

Date: Sept 03, 2002

University of Alberta

Faculty of Graduate Studies and Research

The undersigned certify that they have read and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled **Optimization Based Forest Planning Tools for Sustainable Forest Management** submitted by **David Mateiyenu Nanang** in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Forest Economics.

S Hauer

Dr. G. K. Hauer (Co-Supervisor)

Dr. W. L. Adamowicz (Co-Supervisor)

Dr. G. W. Armstrong

PC Boxall

Dr. P. C. Boxall

Dr. S. Jeffrey (Committee Chair)

Howard Hoganson

Dr. H. Hoganson (External Examiner)

Date: August 19, 2002

Dedication

This thesis is dedicated to:

My Parents: Nanang Nanyun and Jun Konlan-Gbaruk

and

My daughter: Baaloon Sandra Nanang

ABSTRACT

This three-paper thesis applies decomposition techniques to the formulation and solution of large-scale forest management problems. The first paper applied the decomposition procedure to analyze the costs of regulatory constraints and overlapping tenures in Alberta. In general constraints imposed by overlapping tenures led to inefficiencies in wood allocation and substantial increases in the marginal costs of production. Secondly, the effect of the overlapping tenure constraints was unevenly distributed among mills. Consequently, relaxation of the overlapping tenure constraints led to gains for some of the mills and losses for others.

In the second paper, we developed a mixed-integer non-linear programming (MINLP) model that integrates access road development with forest harvest scheduling. Inclusion of road access costs concentrated forest management activities to fewer locations over the planning period compared to when road construction costs were zero. Also, positive access costs reduced the frequency with which locations are accessed during the planning horizon. The model provides important shadow price information on the various constraints in the model that gives insights into future production costs or timber prices, which are valuable for determining supply planning as well as silvicultural and road building investment decisions.

The third paper, which was an extension of the second, incorporated a recreation site choice model into an integrated forest scheduling and access activities model. The results showed that there were significant tradeoffs between timber and non-timber values. The benefits derived by elk hunters were small compared to timber values, and the increase in recreational values due to reduced harvesting could not compensate for the lost timber revenue. Inclusion of non-timber values only slightly affected the forest management schedules and access road development. On the other hand, timber harvesting significantly influenced hunter behavior by concentrating hunters to fewer, unaccessed locations in response to the spreading out of timber harvests on the landscape. The results showed a clear link between landscape characteristics and changes and behavioural responses by hunters.

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CHAPTER 1
INTRODUCTION TO THE THESIS

BACKGROUND

Increasing demands for wood products and for non-timber goods and services in forests place ever greater requirements on scheduling and planning of management activities on forested areas in Canada. The problem is extremely complicated because it requires scheduling the harvest and other management activities on many forest classes over time and space and must simultaneously consider the forest-wide constraints on wood demand (usually from several locations) and other non-timber goods and services. The fact that wood demand usually arises from several locations and from several different companies with different use rights to wood or forest tenures over the same land base complicates planning even further. The purpose of this research was to expand previous forest management scheduling models to: i) consider overlapping tenure constraints, ii) incorporate strategic forest access considerations in a spatial and intertemporal framework, and iii) incorporate a spatial model of non-timber forest user behavior directly into the forest management model's objective function. These objectives add significant complications to the traditional forest-management scheduling problem.

The three forest scheduling problems examined in this thesis are based on mathematical programming techniques. Mathematical programming is widely used in planning the management of forests because of its adaptability to the wide range of problems encountered in forest management. Due to the large spatial and temporal dimensions considered, harvest scheduling usually involves very large linear programming models, including, in some cases, hundreds of thousands of choice variables and thousands of constraints. Current forest management scheduling models used in Alberta for allowable cut calculations, for example, incorporate large amounts of spatial detail, at least for the first harvest in the planning horizon, with some even using the stand polygon as a spatial unit (Messmer 2001 Weyerhaeuser, Edmonton, Alberta, pers. comm.). These models tend to have simple "maximize volume over time" objective functions and tend to be quite limited in terms of the number of alternatives they consider (e.g., the number of regeneration alternatives is usually restricted to one) and typically do not incorporate spatial representation of demand for forest products. Even although the models consider relatively few options given the large spatial detail represented and despite recent advances in computer technology, the resulting models can be cumbersome to use and may take many hours or days to solve.

Until recently, optimization approaches to forest management scheduling had been largely abandoned in forest management planning in Canada in favor of simulation methods. A simulation model has the advantage of being able to incorporate spatial and temporal detail with relative ease. However, one problem with simulation models is that the analyst does not know how close the latest simulation run is to the best solution. In addition, most simulation approaches employ short term scheduling heuristics, which do not lend themselves to intertemporally optimized harvest schedules nor do they provide the shadow price information on any of the constraints incorporated in the formulation.

The introduction of an alternative "simulation approach" by Hoganson and Rose (1984) and the augmented lagrangian method by Gunn and Rai (1987) which are based on dual decomposition techniques are not only based on optimization procedures but are also capable of incorporating a large amount of spatial and temporal detail. These decomposition methods take advantage of the presence of special structure found in mathematical programming problems to break the larger problem down into easier-to-solve subproblems. The models developed in this study use a dual decomposition technique based on the interpretation of the dual side of a linear or non-linear programming formulation of a Model II forest-management scheduling model (Johnson and Scheurman, 1977). The main advantage of this approach is its ability to include a large amount of spatial and temporal detail in the models. A second advantage is that the procedure focuses on shadow prices for the constraints. This means that shadow prices in terms of $/m^3 or $/ha for each of the constraints in the model can be obtained. The shadow prices from the estimated output constraints (output prices) provide useful information. The shapes, positions, as well as slopes of output price curves provide information on management alternatives, rotation ages and profitability of stands to be harvested. For example, a steep and positively sloped curve implies that market prices are rising, and by holding timber longer it will increase in value and thus favor longer rotations (Hoganson and Rose, 1984). The simulation approach also allows for optimal scheduling and balancing of multiple products from pure and mixed forest stands. Because the method optimizes over time and across stands, it is ideal for mixedwood management because demands can be specified for each species and product and optimized simultaneously. Furthermore, it is possible to model multiple markets and demands in different locations. It is also capable of incorporating multiple markets for other goods and services such as multiple demand locations for hunters. Finally, the method is capable of including non-linearities such as binary 0-1 variables for modeling forest access and non-linear product demands.

The first paper in this thesis extends the decomposition procedure by Hoganson and Rose (1984) to analyze the costs of overlapping tenures in Alberta. Overlapping tenures occur when more than one firm has harvesting rights on the same piece of land. Usually, the larger firm enters into a Forest Management Agreement (FMA) with the Province, whilst the smaller firm(s) (quota or commercial timber permit holders) are entitled to percentages of the annual allowable cut (AAC) for specific Forest Management Units (FMUs) within the FMA. The objectives of this paper are to: i) extend the dual decomposition approach to incorporate the types of constraints implied by overlapping tenure, and ii) estimate the costs associated with various constraints implied by overlapping tenures in Alberta.

The second paper extends the simulation approach to model long-term timber supply on the Drayton Valley FMA of Weyerhaeuser Canada with explicit consideration of access. Access is an important issue in forest planning because a large fraction of total management costs is spent on road construction or upgrading. Adequate access must be provided before management activities can be scheduled (Weintraub and Navon, 1976). Furthermore, incorporating access in strategic forest planning models is important because of the cumulative effects of access provision on non-timber values in the forest.

For example, the ease with which hunters access hunting sites, and the welfare of such hunters are determined in part by the level of access within a forest. Consequently, the basic structure of the model in this paper was such that it could be extended to deal with non-timber benefits resulting from hunting within the study area. The main objectives were to: 1) integrate access planning into a large, spatially detailed strategic forest scheduling model, and 2) examine the effects of explicitly including access development costs on the harvests schedule and road development.

Recently, multiple use issues have gained importance and recognition in forestry, and so forest managers are increasingly interested in modeling the trade-offs that would be associated with the provision of wildlife habitats and/or recreation. The third paper deals with the incorporation of recreation choice models into forest scheduling problems. We use mixed-integer non-linear programming (MINLP) to incorporate a spatially explicit utility function for hunter recreation values into a forest level harvest scheduling and access road development model. The resulting behavioral model was used to examine: i) how timber harvests schedules change when non-timber values are included, ii) how the welfare of hunters change with changes in timber harvest, iii) how timber values change in the presence of non-timber benefits, and iv) the effect of timber harvests and access development on hunter behavior.

THE SIMULATION METHOD

The approach used to solve all the mathematical programming problems in this thesis is based on the simulation method by Hoganson and Rose (1984). Hoganson and Rose's approach is similar to the lagrangian relaxation method described by Fisher (1981, 1985), Hauer and Hoganson (1996) and Held *et al.*, (1974). The approach is best illustrated by considering the following Model I formulation of timber management scheduling problem as defined by Johnson and Scheurman (1977):

The Model I formulation may be generalized as:

$$P \qquad Maximize \quad \sum_{t=1}^{T}\sum_{p=1}^{P} R(M_{pt}) - \sum_{i=1}^{I}\sum_{j=1}^{J_i} c_{ij}x_{ij} \qquad (1.1)$$

subject to

$$\sum_{i=1}^{I}\sum_{j=1}^{J_i} v_{ijpt}x_{ij} = M_{pt} \qquad \forall pt \qquad (1.2)$$

$$\sum_{j=1}^{J_i} x_{ij} = A_i \qquad \forall i \qquad (1.3)$$

$$x_{ij} \geq 0 \qquad \forall ij \qquad (1.4)$$

where

A_i = the area in hectares of stand type i present in the initial period

$R(M_{pt})$ = the discounted revenue (as a function of output) for producing output level for product type p in period t

x_{ij} = the area in hectares of stand type i assigned to management sequence j

c_{ij} = the cost of managing stand type i assigned to management sequence j

M_{pt} = the desired output level for product type p in period t

I = the number of stand types

J_i = the number of management sequences for stand type i

v_{ijpt} = the volume per unit area of product type p in period t for analysis area i if management sequence j is followed

T = the length of the planning horizon

P = total number of wood products

Equation (1.1) is the objective function which maximizes the net present value of all management sequences over the planning horizon. The first set of constraints (1.2) forces the desired output levels to be achieved in each period whilst the second set (1.3) defines the initial area in each stand type. The output level constraints are the "key constraints" as they tie the problem together. Without the output level constraints, the problem can be decomposed into independent problems – one for each initial stand type - and each solved separately without using linear programming. The basis of the simulation approach lies in the interpretation of the dual variables of the key constraints (output level constraints). The dual variable for each output level constraint is an estimate of the benefit of producing one additional unit in the corresponding period. Therefore, they are interpreted as the marginal revenue or cost of production. The dual variable for each initial area constraint in the above model is an estimate of the change in the benefits of all desired output levels if one hectare from the corresponding stand type is added to the land base.

The lagrangian of the above primal problem, Z, can be expressed as:

$$D \quad \underset{m_{pt},a_i}{Minimize} \quad Z_D(m,a) \tag{1.5}$$

where $m = (m_{pt}) \forall p,t \quad and \quad a = a_i \ \forall i$ are vectors of shadow prices

and

$$Z_D(m,a) = \underset{x_{ij},M_{pt}}{Max} \sum_{t=1}^{T}\sum_{p=1}^{P} R(M_{pt}) - \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij}x_{ij} + \sum_{p}^{P}\sum_{t}^{T} m_{pt}\left(\sum_{i=1}^{I}\sum_{j=1}^{J} v_{ijpt}x_{ij} - M_{pt}\right)$$
$$+ \sum_{i=1}^{I} a_i\left(A_i - \sum_{i=1}^{I} x_{ij}\right) \tag{1.6}$$

Differentiating the right hand side of (1.6) with respect to x_{ij} and M_{pt} gives first order conditions which can be re-arranged as follows:

$$a_i \geq \sum_{t=1}^{T}\sum_{p=1}^{P} v_{ijpt}m_{pt} - c_{ij} \qquad \forall ij \tag{1.7}$$

$$R'(M_{pt}) = m_{pt} \qquad \forall pt \tag{1.8}$$

$$a_i \ unsigned \qquad \forall i \tag{1.9}$$

$$m_{pt} \ unsigned \qquad \forall p,t \tag{1.10}$$

where

a_i = the dual variable associated with stand type i

m_{pt} = the dual variable associated with the output level constraint for product p in period t

P = the number of product types

T = the number of time periods

If the dual variables on the output level constraints are known, the right hand sides of equation set (1.7) are constants and lower bounds on the a_i variables. Thus the problem can be solved by finding the maximum lower bound among the set of j

constraints for each analysis area *i*. Since each *j*th constraint represents a management sequence, the problem can be although of as finding the management sequence with the highest net present value where the prices used to value the outputs are the marginal revenue (as defined by Equation 1.8) or shadow prices associated with producing the outputs (the m_{pt}'s). Once the optimal management sequence is found for each stand, the output levels for each product type in each period can be determined by summing up the output levels associated with the optimal management sequence for each stand type.

The solution of the model as described by Hoganson and Rose (1984) proceeds with the following steps:

1) Use prior information about the problem to estimate the marginal revenue of production for each output and period (that is, the m_{pt}'s).
2) Assume the m_{pt} estimates are correct and solve formulation *D* for the remaining dual variables (a_i's).
3) Determine the primal solution (the x_{ij}'s) in formulation (*P*) that corresponds to the optimal dual solution. This primal solution is not necessarily a feasible solution.
4) Determine the output levels for the primal solution found in step 3 so that the primal solution can be tested for feasibility.
5) Test for primal feasibility. If the output levels determined in step 4 are close to their desired output levels (M_{pt}'s), stop, the primal solution is both an optimal and a near-feasible solution.

Otherwise:

6) Use the output levels determined in step 4 and a basic understanding of the relationship between output levels and marginal revenue of production to re-estimate the m_{pt} values.
7) Return to step 2.

The most important aspect of the simulation approach is the ability to re-estimate the dual prices using previous price estimates. The various methods of adjusting the prices on all the constraints are discussed in detail in previous studies (e. g., Hoganson and Rose, 1984; Hauer, 1993). The price adjustment procedures for each of the three problems are summarized in Appendix III, IV, and V.

The three problems that are formulated in the three papers of this thesis present a number of extensions to the conventional timber scheduling modeling approach. In Chapter 2, the need to incorporate many types of spatial constraints on forest harvest to model overlapping tenures, and inequality constraints such as spatial harvest constraints and non-declining even flow regulatory constraints result in a very large model. Chapter 3 incorporates forest access, which generates 0-1 binary variables, and in Chapter 4, a utility theoretic spatial choice model is used to represent hunter choice for hunting trips. The enormous number of decision variables and constraints resulting from the substantial spatial and temporal representation in our models prompted the need for a decomposition technique. Moreover the addition of the 0-1 binary integer variables and hunting site choice models increases the need for a decomposition approach to make model solution and/or analysis more feasible.

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CHAPTER 2
A DECOMPOSITION APPROACH TO MODELING OVERLAPPING TENURES IN ALBERTA: A CASE STUDY

INTRODUCTION

Changing technology in wood processing has made it possible to utilize species of wood, which previously had no commercial value. The emergence of these species as valuable resources occurred while traditionally valued species often growing as part of the same forest stand, also increased in value (Luckert, 1991). This situation led to new management problems associated with the joint production of multiple outputs from forestland. The Alberta Forest Service has been allocating harvesting rights to more than one firm on the same piece of land as one way of ensuring that the newly valued resources are utilized (Luckert, 1991). As a result, overlapping tenures have emerged in Alberta as a significant issue as the number of Forest Management Agreement (FMA) areas has increased. Presently, there are 18 FMAs and 92% of all volume quotas in Alberta are embedded in these FMAs.

Overlapping tenures in Alberta impose two main types of constraints on woodland operations. To understand these constraints a short description of overlapping tenures is warranted. Overlapping tenures are areas of land where harvesting rights are allocated to more than one firm on the same piece of land. The usual configuration of overlapping tenures in Alberta is that one firm has an area based tenure with rights to harvest over the whole area of the forest as well as rights to harvest all or most of the tree species in that area. Along with these rights there are obligations to regenerate harvested areas, plan for the sequencing of harvests over the land base, and ensure that harvesting operations are sustainable. The full set of rights and obligations are set out in a Forest Management Agreement (FMA). The term overlapping tenure comes about when, within the FMA areas, there are embedded volume-based tenures which are held by other firms. Although the volume based tenures or quotas are not "area-based" like FMAs there are usually restrictions on which areas within the FMA boundaries that harvests may take place. In addition, volume based tenures are usually species specific. Only certain species may be harvested by the quota holders and FMA holders. To complicate matters further land is usually classified on the basis of the predominant species and the harvesting rights of volume quota holders are usually restricted to the land base for which species specified by the quota predominate. For example, if the volume quota is for conifer the quota holder may be restricted to harvesting from the conifer land base. In some cases quota holders may also have rights to incidental volumes harvested from other land bases.

While the above sets out the main constraints implied by overlapping tenures, these constraints also interact with other regulations. First, regeneration standards are specified to return forest stands to approximately the same species composition that existed before harvest. Hence, conifer stands are regenerated to return to conifer, deciduous to deciduous and mixed to mixed stands. This may in some cases prevent stands from being regenerated most cost effectively from a social perspective. Second, there are often implicit sustainability constraints applied to the land bases, embedded within the larger FMA land base from which quota holders draw their wood supply. Finally, it is usually unclear as to who has rights to increased allowable cuts that may be obtained by increasing silvicultural input into the forest.

Incorporating the constraints implied by overlapping tenures complicates model formulation in several ways. First, to properly represent the overlapping tenure situation more than one demand location must be represented so that the different tenure holders may be modeled. To fully capture the costs of constraints which are spatial both because they are applied over Forest Management Units (FMUs) within an FMA and because of the different demand locations, multiple supply locations must be modeled. The policy to allow overlapping tenures, which seeks to guarantee a supply of timber to quota holders, makes it necessary to test the sustainability and cost of the existing wood supply configuration over a long period of time. Hence, the spatial detail (transport costs, supply and demand locations) must be maintained over the planning horizon. Second, since overlapping tenures impose restrictions on where and on what kind of land class from which wood can be taken, models must keep track of from which land classes and which locations desired volumes are being taken. These temporal and spatial requirements considerably increase the number of decision variables and constraints to be modeled.

In this paper, we demonstrate the utility of the dual decomposition approach by modeling hypothetical, but plausible timber, supply problems that incorporate overlapping tenure constraints. While the timber supply model is formulated for two real land bases we have modified mill demands and constraints slightly. Hence, while the description of the land base and the overall overlapping tenure situation are meant to be realistic, the model implementation is modified enough to be less realistic in terms of the magnitude of the direction of changes, the wood demands and the number of mills represented. The specific objectives of this paper are to: i) extend the dual decomposition approach to incorporate the types of constraints implied by overlapping tenure, and ii) estimate the costs associated with various constraints implied by overlapping tenures in Alberta. The overlapping tenure constraints that are explicitly examined in this paper are harvest location and land base restrictions.

LITERATURE REVIEW

The focus of this section is to provide an overview of previous studies related to overlapping tenures in Alberta. The literature related to decomposition methods of solving large-scale forest management problems is abundant elsewhere and will not be repeated here. The most popular of these decomposition techniques are: Dantzig-Wolfe decomposition method (Dantzig and Wolfe, 1961); Lagrangian relaxation subgradient method (Fisher, 1981; 1985); augmented Lagrangian method (Gunn and Rai, 1987; Gunn *et. al.*, 1988); and a Lagrangian relaxation method using simple forest management heuristics (also known as the simulation method) (Hoganson and Rose, 1984).

Overlapping tenures in Canada generally result from one of two reasons: either the Crown has granted rights to more than one tenure holder, or a tenure holder has chosen to sublease rights to another tenure holder. Luckert (1991) noted that the problems encountered in administering mixedwood stands are quite different in the two cases, although the sharing of responsibilities are similar whether the crown or the tenure holder is responsible for allocating rights. Luckert (1991) investigated the problems that various governments in Canada face in dealing with mixedwood management using economic tools. The objectives of the study were to determine the kinds of tenure arrangement, which provincial governments use to administer mixedwood stands and to identify the types of problems associated with the administration of such stands. The results showed that where the Crown has granted overlapping rights, administrators indicated several common problems. These include: difficulties of coordinating access to forest stands and in matching wood flows with mill requirements; inappropriate harvesting practices which damage unharvested wood and/or sites for reforestation; difficulties with cooperation and/or negotiations between tenure holders because of imbalanced bargaining power. In contrast to these problems, those provinces, which use subleases reported virtually no problems with such arrangements. Although subleasing harvesting rights is permitted in Alberta, Luckert (1991) found that firms made limited use of this option. Because this study was limited to a qualitative analysis of the problems associated with overlapping tenures, the costs implied by these constraints could not be estimated, and therefore, whether the problems identified were quantitatively significant enough to warrant any policy changes remained unknown.

Cumming and Armstrong (2001) used a simulation approach to conduct a quantitative examination of the costs of overlapping tenures and divided land bases in Alberta. The study was done on the Alberta-Pacific Forest Industries Inc. (APFI) Forest Management Agreement (FMA) area (approx. 74,000 km^2 in area), in which 17 quota holders and the APFI pulp mill were modeled. The model used a simulated forest in which the FMA holder and each quota holder are trying to simultaneously and independently satisfy their mill feedstock requirements. The purpose of the study was to compare these existing tenure arrangements with a global policy where a single agent is responsible for forest management and for supplying all mills with timber. The simulation results showed that under the current arrangements, the FMA area is unable to meet the softwood volume demand in many periods, while under the global planning strategy there is no shortfall in timber volume. Also, the difference in delivered wood costs between the two scenarios was about $140 million and the global strategy also resulted in fewer townships being accessed than the present arrangement. The authors concluded that the costs of overlapping tenures and divided land bases are substantial enough to justify a thorough examination of forest policy in Alberta.

Alavalapati and Luckert (1997) modeled the short-run timber supply of quota holders in Alberta in the face of institutional constraints (allowable cut and mill capacity) and fixed stumpage prices using dynamic optimization techniques. The shadow prices of mill processing capacity and allowable cut restrictions were estimated for large, medium and small tenure holders to reflect the different cost structures of different sized firms. The results indicated all categories of quota holders studied incurred substantial costs due to these two institutional constraints, and that simultaneous elimination of both constraints leads to more cost reduction than the combined savings from eliminating each constraint individually. The focus of that study was on quota holders, and therefore did not address the special problems related to overlapping tenures involving FMA holders that are investigated in this study.

METHODS

Model Formulation

Two Weyerhaeuser FMA areas in Edson and Drayton Valley with a total productive forest area of about 550,000 ha spread over approximately 145 townships were used in this study. The spatial unit of analysis was a ¼ township (one township ≈ 10,000 ha or 100 km^2). Mathematically, the timber supply problem can be described by the set of equations given in Equations 2.1-2.7. The formulation is an extension of the Model II formulation given in Johnson and Scheurman (1977). The objective function (Equation 2.1) maximizes the net benefit of timber products from all mills (firms) with rights to timber on the two FMAs in Edson and Drayton Valley subject to the standard Model II age class constraints, demand constraints, and overlapping tenure constraints. The benefits are the net returns of the value of wood products plus the value of ending forest inventory minus costs of regeneration, harvesting, and transportation. The first term in the objective function (Equation 2.1) is the discounted value of wood products.

While our model is general enough to handle downward sloping demand curves, simplified demand curves are used here, which are perfectly inelastic up to a maximum price cap along which demand is perfectly elastic. The second term in Equation 2.1 is the value of the ending inventory, whilst the last term is the costs of regeneration, harvesting, and transportation.

$$Max \sum_{m=1}^{M}\sum_{t=1}^{T}\beta^{t-1}\left[\int_{0}^{M_{mt}} D_{mt}(X)dX\right]+\sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{t=1}^{T}E_{ins}w_{ins}-\sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{s=-T^o}^{t-z}\sum_{t=1}^{T}c_{ijst}x_{ijst} \tag{2.1}$$

subject to:

Forest Dynamics Constraints:

$$\sum_{j=1}^{J_i}\sum_{t=1}^{T}x_{ijst}+\sum_{j=1}^{J_i}w_{ins}=A_{is} \qquad \forall i,s=-T^o,\ldots,0 \tag{2.2}$$

$$\sum_{j=1}^{J_i}\sum_{h=t+z}^{T}x_{ijth}+\sum_{j=1}^{J_i}w_{ijt}-\sum_{j=1}^{J_i}\sum_{s=-T^o}^{t-z}x_{ijst}=0 \qquad \forall i,t=1,\ldots,T \tag{2.3}$$

Land base Regeneration Restrictions:

$$\sum_{j\in J_i^c}\left(\sum_{t=s+z}^{T}x_{ijst}+w_{ijs}\right)\geq\delta^c\sum_{j=1}^{J_i}\left(\sum_{t=s+z}^{T}x_{ijst}+w_{ijs}\right) \qquad \forall i\in I^c,I^{cd},I^{dc},\forall s=0,\ldots,T-z \tag{2.4}$$

$$\sum_{j\in J_i^d}\left(\sum_{t=s+z}^{T}x_{ijst}+w_{ijs}\right)\geq\delta^d\sum_{j=1}^{J_i}\left(\sum_{t=s+z}^{T}x_{ijst}+w_{ijs}\right) \qquad \forall i\in I^d \quad \forall s=0,\ldots,T-z \tag{2.5}$$

Allowable Harvest Area and Land base Restrictions:

$$\sum_{i\in I^m}\sum_{j\in J_i^m}v_{ijstm}x_{ijst}\leq 0 \qquad \forall s,t \quad and \quad m \tag{2.6}$$

Market Clearing Restrictions:

$$\sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{s=-T^o}^{t-z}v_{ijstm}x_{ijst}\leq M_{mt} \qquad \forall t,m=1,..,M \tag{2.7}$$

$$x_{ijst}\geq 0 \qquad \forall ijst$$

$$w_{ins}\geq 0 \qquad \forall ins$$

$$M_{pt}\geq 0 \qquad \forall pt$$

where the following variables are defined as:

E_{ins} = the discounted net value per unit area of managing stand type *i* with regeneration prescription *n*, starting in period *s* and leaving the stand type as ending inventory

w_{ijs} (w_{ijt}) = area managed stand type *i* with regeneration prescription *j*, in period *s* (period *t*) and left as ending inventory

x_{ijst} (x_{ijsh}) = area managed on stand type *i* with regeneration prescription and market shipping plan *j*, starting in period *s* and final harvest in period *t* (period *h*).

The parameters in the model are defined as:

$D_{mt}(X)$ = the inverse demand function for wood products from mill *m* in period *t*.

c_{ijst} = the discounted cost per unit area of managing stand type *i* with regeneration prescription *j* , starting in period *s* and final harvest in period *t*.

m = counter for mills; $m = 1,...,M$;

$m = 1,...,\overline{m}$ for Weyerhaeuser mills, where $\overline{m} < M$

A_{is} = the number of area unit of stand type *i* in the first period that were regenerated in period *s*.

v_{ijstm} = the volume per unit of wood products from mill *m*, in period *t*, when stand type *i* is regenerated in period *s* and managed with prescription and market shipping plan *j*.

M_{mt} = output of mill *m* in period *t*

δ^c = percentage of analysis area that must be regenerated to conifer species

δ^d = percentage of analysis area that must be regenerated to deciduous species

J_i = the set of regeneration prescriptions and transport destinations for analysis area *i*.

J_i = {(1,1),.....,(N_i,1);........;(1,D),.....(N_i, D)}. Each pair refers to a prescription and destination combination; where N_i is the number of prescriptions for stand *i* and *D* is the number of destinations. It should be noted that wood from any stand *i* can be sent to more than one destination.

J_i^c = subset of J_i that includes regeneration prescriptions that meet the conifer standards

J_i^d = subset of J_i that includes regeneration prescriptions that meet the deciduous standards

I^c, I^d, I^{dc}, I^{cd} = set of conifer, deciduous, deciduous/conifer, and conifer/ deciduous land bases, respectively.

I = the number of stand types

L = number of supply locations

I^m = the set of locations or forest types that are not available to mill m

J_i^m = the set of transport/prescriptions that are defined for mill m

z = minimum time between regeneration and harvest

T = the number of planning periods in the planning horizon

β = discount factor (using 5% discount rate).

T^0 = the number of periods before the first period that the oldest age class of stands were regenerated.

Equation set (2.2) accounts for the forest area regenerated before the planning period (existing stands). Total area harvested during the planning horizon plus area left as ending inventory (at the end of the planning horizon) should equal the initial area (regenerated in period s before planning period). The area regenerated during the planning period is accounted for by Equation (2.3). The equation implies that the total area harvested during the planning period plus area left as ending inventory at the end of the planning period should equal area regenerated during the planning period. Therefore, Equation (2.3) ensures that all harvested areas are regenerated. Current provincial regulations require that conifer land bases be regenerated to at least 80% conifer and deciduous land bases to at least 80% deciduous species. Constraints (2.4) and (2.5) represent these additional regulations. In Equations (2.4) and (2.5), cd (conifer/deciduous) and dc (deciduous/conifer) refer to stands that are defined in the Alberta Regeneration Survey Manual (Alberta Environment, 2000) as mixedwood stands. Equation (2.6) represents restrictions on FMUs and land bases where quota holders are allowed to harvest. When applied to restrictions on the FMUs, Equation (2.6) says that wood cannot flow from locations within FMUs that are restricted from mill m in period t. Equation (2.6) is interpreted similarly when applied to land base restrictions. Equations (2.7) implies that volume of wood products produced from all stands managed in the two FMAs should be less than or equal to the mill demands.

While Equations (2.1) through (2.7) set out the economic problem, the enormous size of the formulation will be difficult to solve, especially if a non-linear demand system is specified. We therefore form the lagrangian and derive the dual variables. These dual variables are important for two main reasons. First, the solution approach of Hoganson and Rose (1984), which is used to solve this model, relies on a direct interpretation of the dual variables. Another rationale for deriving the dual variables is that these variables provide insights and relationships to other literature that are apparent from analysis of the dual, for example the Faustmann (1849) optimal forest rotation model.

The shadow prices on the constraints specified in Equation 2.1 can be determined by first specifying the lagrangian to the primal problem as:

$$
\begin{aligned}
L = {} & \sum_{m=1}^{M}\sum_{t=1}^{T}\beta^{t-1}\left[\int_0^{M_{mt}} D_{mt}(X)dX\right] + \sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{t=1}^{T} E_{ins}w_{ins} - \sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{s=-T^o}^{t-z}\sum_{t=1}^{T} c_{ijst}x_{ijst} \\
& + \sum_{i=1}^{I}\sum_{s=-T}^{0} a_{is}\left[A_{is} - \sum_{j=1}^{J_i}\sum_{t=1}^{T} x_{ijst} - \sum_{j=1}^{J_i} w_{ijs}\right] + \sum_{i=1}^{I}\sum_{t=1}^{T} a_{it}^{r}\left[\sum_{j=1}^{J_i}\sum_{s=-T^0}^{t-z} x_{ijst} - \sum_{j=1}^{J_i}\sum_{h=t+z}^{T} x_{ijth} - \sum_{j=1}^{J_i} w_{ijt}\right] \\
& + \sum_{i=1}^{I}\sum_{s=-T^o}^{t-z} a_{is}^{c}\left[\sum_{j\in J_i^c}\left(\sum_{t=s+z}^{T} x_{ijst} + w_{ijs}\right) - \delta^c\sum_{j=1}^{J_i}\left(\sum_{t=s+z}^{T} x_{ijst} + w_{ijs}\right)\right] \\
& + \sum_{i=1}^{I}\sum_{s=-T^o}^{t-z} a_{is}^{d}\left[\sum_{j\in J_i^d}\left(\sum_{t=s+z}^{T} x_{ijst} + w_{ijs}\right) - \delta^d\sum_{j=1}^{J_i}\left(\sum_{t=s+z}^{T} x_{ijst} + w_{ijs}\right)\right] + \sum_{t=1}^{T}\sum_{s=-T^o}^{0}\sum_{m=1}^{M}\lambda_{tm}\left[-\sum_{i\in I^m}\sum_{j\in J_i^m} v_{ijstm}x_{ijst}\right] \\
& + \sum_{t=1}^{T}\sum_{m=1}^{M} m_{mt}\left[\sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{s=-T^o}^{t-z} v_{ijstm}x_{ijst} - M_{mt}\right] \qquad (2.8)
\end{aligned}
$$

The first order conditions of the lagrangian with respect to x_{ijst}, w_{ins}, and M_{mt}, respectively can be re-arranged as:

$$a_{is} \ge a^r_{it} + a^c_{is}(1-\delta^c) + a^d_{is}(1-\delta^d) + \sum_{m=1}^{M} v_{ijstm}(m_{mt} - \lambda_{tm}) - c_{ijst} \qquad (2.9)$$

$$s = -T^0,\ldots,0;\ t = 1,\ldots,T$$

Equation (2.9) implies that prescriptions that lead to conifer stands will have a subsidy equal to $a^c_{is}(1-\delta^c)$ and those that do not lead to conifer will have a "tax" of $-\delta^c a^c_{is}$. In addition, there is a penalty of λ_{tm} applied to a mill that takes wood from areas where the mill is not allowed to harvest. The conditions under which the right hand side shadow prices (a^c_{is} and λ_{tm}) in Equation 2.9 are present or absent in the bareland values of the existing stands are given in Equations 2.9a to 2.9d.

$$a_{is} \ge a^r_{it} + a^c_{is}(1-\delta^c) + \sum_{m=1}^{M} v_{ijstm}(m_{mt} - \lambda_{tm}) - c_{ijst}$$

$$\forall i \in I^c or\ I^{cd} or\ I^{dc}\ \ \forall i \in I^m\ \ j \in J^c_i\quad s = -T^0,\ldots,0; t = 1,\ldots,T \qquad (2.9a)$$

$$a_{is} \ge a^r_{it} + a^c_{is}(1-\delta^c) + \sum_{m=1}^{M} v_{ijstm} m_{mt} - c_{ijst}$$

$$\forall i \in I^c or\ I^{cd} or\ I^{dc}\ \ \forall i \notin I^m\ \ j \in J^c_i\quad s = -T^0,\ldots,0; t = 1,\ldots,T \qquad (2.9b)$$

$$a_{is} \ge a^r_{it} - \delta^c a^c_{is} + \sum_{m=1}^{M} v_{ijstm}(m_{mt} - \lambda_{tm}) - c_{ijst}$$

$$\forall i \in I^c or\ I^{cd} or\ I^{dc}\ \ \forall i \in I^m\ \ j \notin J^c_i\quad s = -T^0,\ldots,0; t = 1,\ldots,T \qquad (2.9c)$$

$$a_{is} \ge a^r_{it} + a^c_{is}(-\delta^c) + \sum_{m=1}^{M} v_{ijstm} m_{mt} - c_{ijst}$$

$$\forall i \in I^c or\ I^{cd} or\ I^{dc}\ \ \forall i \notin I^m\ \ j \notin J^c_i\quad s = -T^0,\ldots,0; t = 1,\ldots,T \qquad (2.9d)$$

$$a^r_{is} \ge a^r_{it} + a^c_{is}(1-\delta^c) + a^d_{is}(1-\delta^d) + \sum_{m=1}^{M} v_{ijstm}(m_{mt} - \lambda_{tm}) - c_{ijst} \qquad (2.10)$$

$$s = 1,\ldots,T-z;\ t = s+z,\ldots,T$$

The different combinations of subsidies and penalties applied to the bareland values of the regenerated stands are given in Equations 2.10a to 2.10d.

$$a_{is}^{r} \geq a_{it}^{r} + a_{is}^{c}(1-\delta^{c}) + \sum_{m=1}^{M} v_{ijstm}(m_{mt} - \lambda_{tm}) - c_{ijst}$$

$$\forall i \in I^{c} or\ I^{cd} or\ I^{dc}\ \forall i \in I^{m}\ j \in J_{i}^{c} \quad s = 1,...,T-z; t = s+z,...,T \qquad (2.10a)$$

$$a_{is}^{r} \geq a_{it}^{r} + a_{is}^{c}(1-\delta^{c}) + \sum_{m=1}^{M} v_{ijstm} m_{mt} - c_{ijst}$$

$$\forall i \in I^{c} or\ I^{cd} or\ I^{dc}\ \forall i \notin I^{m}\ j \in J_{i}^{c} \quad s = 1,...,T-z; t = s+z,...,T \qquad (2.10b)$$

$$a_{is}^{r} \geq a_{it}^{r} - \delta^{c} a_{is}^{c} + \sum_{m=1}^{M} v_{ijstm}(m_{mt} - \lambda_{tm}) - c_{ijst}$$

$$\forall i \in I^{c} or\ I^{cd} or\ I^{dc}\ \forall i \in I^{m}\ j \notin J_{i}^{c} \quad s = 1,...,T-z; t = s+z,...,T \qquad (2.10c)$$

$$a_{is}^{r} \geq a_{it}^{r} + a_{is}^{c}(-\delta^{c}) + \sum_{m=1}^{M} v_{ijstm} m_{mt} - c_{ijst}$$

$$\forall i \in I^{c} or\ I^{cd} or\ I^{dc}\ \forall i \notin I^{m}\ j \notin J_{i}^{c} \quad s = 1,...,T-z; t = s+z,...,T \qquad (2.10d)$$

$$a_{is} \geq E_{ins} + a_{is}^{c}(1-\delta^{c}) + a_{is}^{d}(1-\delta^{d}) \qquad s = -T^{0},...,0 \qquad (2.11)$$

The conditions under which the subsidies or taxes apply to Equation 2.11 are given in Equations 2.11a and 2.11b.

$$a_{is} \geq E_{ins} + a_{is}^{c}(1-\delta^{c}) \quad \forall i \in I^{c} or\ I^{cd} or\ I^{dc}\ \forall j \in J_{i}^{c} \quad s = -T^{0},...,0 \qquad (2.11a)$$

$$a_{is} \geq E_{ins} - \delta^{c} a_{is}^{c} \quad \forall i \in I^{c} or\ I^{cd} or\ I^{dc}\ \forall j \notin J_{i}^{c} \quad s = -T^{0},...,0 \qquad (2.11b)$$

$$a_{is}^{r} \geq E_{ins} + a_{is}^{c}(1-\delta^{c}) + a_{is}^{d}(1-\delta^{d}) \qquad s = 1,...,T \qquad (2.12)$$

The conditions under which the subsidies or taxes apply to Equation 2.11 are given in Equations 2.12a and 2.12b.

$$a_{is} \geq E_{ins} + a_{is}^{c}(1-\delta^{c}) \quad \forall i \in I^{c} or\ I^{cd} or\ I^{dc}\ \forall j \in J_{i}^{c} \quad s = 1,...,T \qquad (2.11a)$$

$$a_{is} \geq E_{ins} - \delta^{c} a_{is}^{c} \quad \forall i \in I^{c} or\ I^{cd} or\ I^{dc}\ \forall j \notin J_{i}^{c} \quad s = 1,...,T \qquad (2.11b)$$

$$\beta^{t-1} D'_{mt}(X) = m_{mt} \qquad \forall mt \qquad (2.13)$$

The right hand side of Equation (2.9) can be interpreted in terms of costs and benefits. The first term represents the value of the bareland after harvest. The second two terms represent adjustments to the land value due to regenerating stands to conifer and hardwood species respectively. These 'subsidies' depend on the proportions of the land base regenerated to either conifer or deciduous species. There is also a penalty of λ_{tm} applied if a mill takes wood from areas where the mill is not allowed to harvest. The different combinations of conifer subsidies and penalties applied to bareland values of the existing stands are given in Equations 2.9a to 2.9d. The interpretations of Equations 2.9a to 2.9d are quite straightforward. For example, Equation 2.9a represents a situation where a mill takes wood from a location it is not allowed to harvest (thereby violating Equation 2.6 and incurring a penalty of λ_{tm}) and also regenerates the stands to conifer (and gets the subsidy associated with regeneration to conifer of $a_{is}^{c}(1-\delta^{c})$. Equations 2.10a to 2.10d for the regenerated stands are interpreted similarly to Equations 2.9a to 2.9d. The fourth term in Equation 2.9 represents the value of wood products sold, adjusted for restrictions in the destinations as specified by constraints Equation (2.6). The last term represents the management and transportation costs. Therefore, a_{is} (the left-hand side of 2.9) represents the net present value of the land value for the initial stand i of age s (i. e., the value of the existing stands). Equation (2.10) follows a similar interpretation as Equation (2.9). The right hand side of this equation can be interpreted as net benefits from the alternative prescriptions plus the value of future rotations starting with bareland in period s and harvesting in period t. Equation (2.11) means the bareland value of the existing stand for any analysis area is at least as great as its value if left as ending inventory and the extra benefits of regenerating the stands to conifer and deciduous species. In Equation 2.11a, there is a subsidy of $a_{is}^{c}(1-\delta^{c})$ for existing stands regenerated to conifer and a tax of $-\delta^{c}a_{is}^{c}$ in Equation 2.11b if the stands are not regenerated to conifer. Equation (2.12) implies the bareland value of the regenerated stand for any stand type is at least as great as its value if left as ending inventory. In Equation 2.12a, there is a subsidy of $a_{is}^{c}(1-\delta^{c})$ for regenerated stands that are regenerated to conifer and a tax of $-\delta^{c}a_{is}^{c}$ in Equation 2.12b if the stands are not regenerated to conifer. The final first order condition (Equation 2.13) has a straightforward interpretation, and means that the discounted revenue received from the sale of an extra unit of wood product from mill m in period t equals the price of the same output in period t.

Data Descriptions

This section briefly describes the data and their sources, as well as methods used to derive some of the variables. As indicated earlier, the objectives of the study are to investigate forest management scheduling problems in Alberta, using the above-specified model. To accomplish this task, we need to classify the two FMA areas into stands, project the growth and yield of these stands, determine the value of ending inventory, compute the soil expectation values of the stands for the different prescriptions, and schedule the various stands for harvesting. These procedures required large amounts of data from many different sources. The different types of data include inventory, growth and yield, transport costs, mill locations and outputs, shares of allowable cut for the various firms, regeneration prescriptions, etc. These data types are briefly described below together with their sources. Where applicable, the data described in this section is used throughout the thesis. In chapters requiring extra data, these are described in the appropriate chapters.

Forest type classification

The total area of the two FMAs was classified into forest types (or biotypes) based on the 1986 Alberta Vegetation Inventory (AVI). Forest types are defined based on the cover type (conifer, deciduous, conifer deciduous, or deciduous conifer), dominant species, and the timber productivity rating. Cover type measures species composition of the stands based on crown closure. Species composition in the AVI shows the percentage of each species to the nearest 10%. The AVI identifies five timber productivity ratings (TPR). The TPR is the potential timber productivity of a stand based on height and age of dominant and co-dominant trees of the leading species. The four TPR codes G, M, F, and U are interpreted as good, medium, fair and unproductive sites. Based on this classification, there were 32 forest types. The initial age class distributions for the two FMA areas are given in Appendix II, Tables A5 and A6. Because transportation costs play an important role in this study, each forest type and age combination was further divided into supply location/forest type/age class combination. Therefore, a supply location and forest type combination will be referred to as a stand type, whilst a stand type and age class combination will be called an analysis area.

Regeneration prescriptions

Two types of regeneration treatment prescriptions were defined for existing stands and regenerated stands, according to forest type. For existing stands, only one tending operation was prescribed. Three prescriptions were assigned to regenerated (bare land) stands. These were natural regeneration, basic planting and planting with tending operations (Intensive silviculture). Natural regeneration involves allowing the stands to regenerate naturally with little intervention by the firm and is assumed to cost nothing. Basic planting involves planting of seedlings with very little tending operations and costs $1000/ha. The most intensive prescription is the planting with tending. The tending operations considered are herbicide applications, spacing of stands, and commercial thinning where necessary, and these were assumed to cost $1450/ha. All conifer and deciduous land bases were prescribed to regenerate into conifers and hardwoods respectively.

Growth and yield

For each analysis area and regenerated stands, net merchantable volumes for the three species types were projected using yield curves developed by staff of Weyerhaeuser Company. The three species types are pine, spruce (white and black), and aspen. Yield curves are developed for each cover type, dominant species, timber productivity rating, and crown density. Crown density is classified in the Alberta Vegetation Inventory (AVI) into four groups; A, B, C and D, from the lowest to the highest density. Crown closure measures the percentage of ground area covered by the vertical projection of tree crowns onto the ground. In terms of percentages of crown cover, the codes A, B, C and D represent respectively, 6-30%, 31-50%, 51-70% and 71-100%. Based on this classification, there were 128 yield curves for the two FMAs. The yield equations are given in Appendix II, Table A7. Since tree size affects both processing and harvesting costs, it was considered important to sort tree products. Product sorting was limited to two classes: sawtimber and merchantable volumes. Therefore, there were six tree product types used in this study.

Harvesting, transportation, and milling costs

Transportation cost of wood from each supply location to each demand location was calculated along the shortest distance possible in the road network. The shortest distances were calculated from the center of each supply location to each mill. Harvesting costs per cubic metre was estimated based on the tree-to-truck cost equation developed by Beck et al. (1987). The resulting harvests costs by age are given in Appendix II, Table A3. This method of determining harvesting costs recognizes that costs decrease with increasing tree age (size). Therefore, a variable cost structure was used for each stand type. A constant harvesting cost for all ages will tend to favor shorter rotations. Transportation costs from supply location to mill was estimated at \$0.03/km/m^3 based on a study by Beck *et al.* (1987). The data for milling costs and conversions from roundwood to lumber, OSB, and chips for the various mills in the model are given in Appendix II, Table A4.

Model Scenarios and Overlapping Tenure Constraints

In this section we outline the eight hypothetical scenarios examined in this paper. The first scenario, which is called the Baserun is meant to represent a case where there are no overlapping tenure restrictions. The Baserun and Scenario 1 are designed to reveal the effect of restrictions on where tenure holders may harvest. In Scenario 1, the overlapping tenure constraints described in Table 2.1 are present. In Scenario 1a, the land base restriction (LBR) is removed, leaving only the harvest location restriction. Scenario 1b drops the harvest area/location restriction (HLR) and examines the effect of the land base restrictions only. Scenario 2 is similar to the Baserun, except that the annual allowable cut for conifer for the two FMAs has been increased by approximately 20,000m^3/year, with all increased AAC allocated to the FMA holder. The AAC increase for deciduous species is 25,000m^3/year for the Drayton Valley FMA and 20,000 m^3/year for the Edson FMA. In Scenario 3, the increased AAC is distributed proportionately to all tenure holders, based on their shares of the AAC for each FMA. Scenarios 4 and 5 use the same AAC allocations as in Scenario 2 and 3 respectively, but Scenarios 4 and 5 drop the harvest location and land base restrictions. None of these scenarios is meant to represent exactly what is actually happening on the two FMA areas. Rather the scenarios are meant to be representative of the types of constraints found when overlapping tenures are present on this area or other FMA areas.

Table 2.1. Summary of mill target demands and overlapping tenure constraints.

	Types of constraints					
	Demand (Eqn. 2.7)				Restrictions on areas that mills allowed to harvest (Eqn. 2.6)	
Model run	Demand Locations	Max Mill price $\$/m^3$ *	Max 000s m^3/yr	Wood type	Allowed locations	Allowed stand types
BaseRun	1 Sawmill (DV)	200	220	SW	All	All
	2 Sawmill (ED)	200	20	SW		
	3 Sawmill (WC)	200	20	SW		
	4 OSB mill (DV)	100	215	HW		
	5 OSB mill (ED)	100	200	HW		
	6 Pulp mill (WC)	80	100	SW		
Scenario 1	1 Sawmill (DV)	100	220	SW	Drayton, All	SW, HW
					Edson, E1, E2, W6	SW, HW
	2 Sawmill (ED)	100	20	SW	Edson, E1,E2	SW
	3 Sawmill (WC)	100	20	SW	Edson, W6	SW
	4 OSB mill (DV)	100	215	HW	All	HW, SW
	5 OSB mill (ED)	100	200	HW	All	HW, SW
	6 Pulp mill (WC)	80	100	SW	Edson, W6	SW
Scenario 1a	1 Sawmill (DV)	200	220	SW	Drayton, All	All
					Edson, E1, E2, W6	
	2 Sawmill (ED)	200	20	SW	Edson, E1, E2	
	3 Sawmill (WC)	200	20	SW	Edson, W6	
	4 OSB mill (DV)	100	215	HW	All	
	5 OSB mill (ED)	100	200	HW	All	
	6 Pulp mill (WC)	80	100	SW	Edson, W6	

Scenario 1b	1 Sawmill (DV)	200	220	SW	All	SW, HW
	2 Sawmill (ED)	200		SW		SW
	3 Sawmill (WC)	200	20	SW		SW
	4 OSB mill (DV)	100	20	HW		HW, SW
	5 OSB mill (ED)	100	215	HW		HW, SW
	6 Pulp mill (WC)	80	200	SW		SW
			100			
Scenario 2	1 Sawmill (DV)	200	240	SW	Drayton, All	SW, HW
					Edson, E1, E2, W6	SW, HW
	2 Sawmill (ED)	200	20	SW	Edson, E1, E2	SW
	3 Sawmill (WC)	200	20	SW	Edson, W6	SW
	4 OSB mill (DV)	100	240	HW	All	HW, SW
	5 OSB mill (ED)	100	220	HW	All	HW, SW
	6 Pulp mill (WC)	80	100	SW	Edson, W6	SW
Scenario 3	1 Sawmill (DV)	200	230	SW	Drayton, All	SW, HW
					Edson, E1, E2, W6	SW, HW
	2 Sawmill (ED)	200	22	SW	Edson, E1, E2	SW
	3 Sawmill (WC)	200	22	SW	Edson, W6	SW
	4 OSB mill (DV)	100	240	HW	All	HW, SW
	5 OSB mill (ED)	100	220	HW	All	HW, SW
	6 Pulp mill (WC)	80	110	SW	Edson, W6	SW
Scenario 4	1 Sawmill (DV)	200	240	SW	All	All
	2 Sawmill (ED)	200	20	SW		
	3 Sawmill (WC)	200	20	SW		
	4 OSB mill (DV)	100	240	HW		
	5 OSB mill (ED)	100	220	HW		
	6 Pulp mill (WC)	80	100	SW		
Scenario 5	1 Sawmill (DV)	200	230	SW	All	All
	2 Sawmill (ED)	200	22	SW		
	3 Sawmill (WC)	200	22	SW		
	4 OSB mill (DV)	100	240	HW		
	5 OSB mill (ED)	100	220	HW		
	6 Pulp mill (WC)	80	110	SW		

* Note: This is cubic meters of final product. Also, SW=conifer and HW=deciduous; A capital letter followed by a number (e. g., E1) in the second but last column refers to the various Forest Management Units in the FMAs.

The second column in Table 2.1 identifies the demand location of which there are six. The demand locations are Drayton Valley (DV), Edson (ED), and Whitecourt (WC). The demand locations also specify the type of demands. In this example there are 3 types of demand locations (Sawmills, OSB mills, and Pulp mills). The third column shows an estimate of the maximum price in terms of $\$/m^3$ of final product that could be paid at the mill gate. These maximum price levels were set based on current estimates of the prices of lumber and OSB. For the pulp mill the final product is defined as an intermediate product, wood chips. The fourth column shows the maximum volumes that can be consumed by each mill on an annual basis (the right hand side of Equation 2.7). A combination of the maximum prices and volumes describe the demand curves for the various wood products. In this set of runs demands for hardwoods and softwoods add up approximately to the total volumes of these species harvested on the Edson and Drayton Valley FMA areas during the last 5 years. Hence, the maximum volumes harvest-able in each area represent a combination of mill capacities and allowable cuts. In fact, the maximum volumes add up approximately to the allowable cuts for hardwood and softwoods. For this reason we do not impose allowable cut constraints because these constraints are redundant or nearly so in the presence of the maximum harvest levels shown in the fourth column of Table 2.1. Also, the maximum volumes shown in Table 2.1 are determined based on the percentage shares that each mill is allocated under overlapping tenure agreements. Another constraint in the model is the restriction that conifer land bases must be regenerated back to conifer, and deciduous stands back to deciduous species (Equations 2.4 and 2.5). These constraints were implemented in the model by restricting the available prescriptions in the data files, rather than imposing the constraints in the model with all available regeneration prescriptions.

The last two columns in Table 2.1 summarize the overlapping tenure constraints (Equation 2.6). The second to last column shows the Forest Management Units (FMUs)[1] from which each mill is allowed to harvest while the last column shows the cover types from which each mill is allowed to harvest. The former restriction will be referred to as the harvest location restriction (HLR), whilst the latter will be called the land base restriction (LBR). The land base restriction can be viewed as a weaker restriction than the harvest location constraint. This is because the harvest location constraint excludes all stands in a location from a mill, whilst the land base restriction excludes a mill from harvesting conifer species on mixedwood land bases in the locations they have rights to harvest. The sawmill in Drayton Valley and the two OSB mills represent the FMA tenure holder's demand locations. The area restrictions for these mills limit harvests so that wood may be harvested only from the FMA that is tied to the mill. Hence, the sawmill in Drayton Valley may take wood from Drayton Valley FMA and a percentage of the conifer volume from Edson in the FMUs specified in Table 2.1. However, the sawmills and the pulp mill may harvest wood only from Edson. In addition, they are limited to harvest only from certain areas or FMUs within the Edson FMA and from the conifer cover type. In some scenarios, these constraints are eliminated entirely so that wood may flow to any mill from any location. In such scenarios wood is optimally allocated on the basis of maximizing net returns.

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[1] In general, a Forest Management Unit (FMU) is a subunit of a larger Forest Management Agreement (FMA) area.

Model Size

The major drawback of the problem defined above is its extremely large size. The total number of analysis areas (forest types/locations/age class combinations) is 29,885 for this management problem with 16,828 in Edson and the remaining 13,057 in Drayton Valley. We define the number of decision variables and constraints to the overlapping tenure problem using the following assumptions:

1. a planning horizon of 100 years with 10 planning periods (10 years per planning period)
2. a minimum rotation of 40 years
3. three regeneration prescriptions per stand (natural regeneration, basic, and intensive silviculture)
4. a total of 6,741 stand types
5. approximately 10 shipping alternatives for each stand. This is based on 3 species with 2 size classes for each species, 4 possible destinations for softwood species, two possible destinations for hardwood species and the assumption that only half of these alternatives would be available for each stand on average.

Table 2.2. Calculation of the number of constraints for the overlapping tenures model.

Eqn No.	Constraint type	Constraint calculation		Number of constraints
1	Sawmill demand	3 sawmills	x10 periods	30
2	OSB mill demand	2 OSB mills	x10 periods	20
3	Pulp mill demand	1 pulp mill	x10 periods	10
4	Sawmill chip production	3 sawmills	x10 periods	30
5	Initial area constraints	29,885 analysis areas		29,885
6	Area harvested = area regenerated	6741 stand types	x10 periods	67,410
7	Harvest location and land base restrictions*	2013 harvest location and land base restrictions	x10 periods	20130
Total				117,515

*Note: There are a total of 2013 locations restricted to all six mills. These are made up of 1866 harvest location restrictions and 238 land base restrictions.

Table 2.3. Calculation of the number of decision variables for the overlapping tenures problem.

Variable Types	Birth period	Number of periods	Number of shipping alternatives	Number of prescriptions	Number of stand types or analysis areas	Number of decision variables
Initial Stands						
Harvesting variables		6	10	1	29,885	1,793,100
Ending inventory		1	-	1	29,885	29,885
Regeneration stands						
Harvest and regeneration variables	1	(10-4*-1**)	10	3	6741stand types	1,011,150
	2	(10-4-2)	10	3	6741stand types	808,920
	3	(10-4-3)	10	3	6741stand types	606,690
	4	(10-4-4)	10	3	6741stand types	404,460
	5	(10-4-5)	10	3	6741stand types	202,230
	6	0				
	7	0				
	8	0				
	9	0				
	10	0				
Ending inventory		10		3	6741stand types	202,230
Total						5,058,665

Note: * Minimum age between harvests, **Birth period

With these assumptions, the resulting model has approximately 5 million decision variables and about 118 thousand constraints (Tables 2.2 and 2.3). The extremely large size of the problem led to the adoption of a dual approach in order to solve the model.

Solution Approach

The model was solved using a variant of the dual decomposition algorithm proposed by Hoganson and Rose (1984). The detailed algorithm for implementing this model is given in Appendix III. The principles behind this method are extremely simple. Using duality theory from mathematical programming, the original linear programming formulation can be viewed as a series of individual stand level decision problems where the stand level decisions include harvest timing for initial and subsequent harvests, mill destination for each timber type, and regeneration options. All the possible stand level decisions are evaluated with a stand level objective function that is linked to the forest level objectives via shadow prices on the forest wide constraints. The solution to the stand level problem amounts to a stand level benefit-cost analysis. Costs include harvest, regeneration and transport costs. Benefits include the marginal value of timber derived from the forest level demand constraints. Other costs include shadow prices or marginal costs of forest wide constraints that affect the stand of interest. The stand level decision problems are extremely easy to solve using dynamic programming. The solution is the combination of rotation, regeneration, and transport decisions that yield the highest net present value. The algorithm begins by solving each stand level problem using initial guesses at the shadow prices for each forest wide constraint. After all the stand level problems are solved the volume flows implied by the harvest timing and transport options are added up and compared to the demand and constraint levels. If the flows deviate from the constraint levels and mill demand levels then the shadow prices are adjusted using simple intuitive shadow price adjustment procedures. For example, if the harvest area restriction for a mill is violated (that is wood is delivered from a supply area to a mill when the mill is not allowed to harvest from that area) then the shadow price on the constraint is increased. This has the effect of imposing a cost penalty on transport options that violate the harvest area restriction. A second example is when wood is oversupplied to a mill according to the mill demand constraint then the shadow price on the mill constraint will be decreased. Once the shadow prices have been adjusted the stand level problems are solved again. This process is continued until the flows converge and all constraints are satisfied within a reasonable tolerance and there is no systematic deviation of constraints over time. The most important aspect of this approach is the ability to re-estimate the dual prices using previous price estimates. The various methods of adjusting the prices on the constraints are discussed in detail in previous studies (e. g., Hoganson and Rose 1984, Hauer 1993).

RESULTS

Model Performance

In order to implement the model runs, initial price estimates were given for each of the three end products for each demand location. All models were run on a microcomputer with a Pentium III 500 Mhz microprocessor. The criteria set for determining when to stop a run were based on the average absolute percentage deviation of the end product from the target demand for each mill, the number of locations violating the harvest location and land base restrictions, and the difference between the objective and lagrangian function values. Each iteration of the model takes about 5 seconds and it takes about 350 iterations for shadow prices and objective function values to converge. Hence, the model takes about 30 minutes to arrive at a solution[2].

Figures 2.1 and 2.2 show the simulated outputs for the final iteration and demand targets for the sawmills and OSB mills for the Baserun. The graphs show that the flows are not only close to the final demands, but are also randomly distributed around the demands. It should be noted that the demand (solid) lines in the simulated output graphs are the vertical portions of the demand curves. Demands may be satisfied if the prices reach the maximum levels given in Table 2.1 and the volumes are less than the vertical portions of the demand curves. The average absolute deviations of the output from the mill demands, the number of locations violating the harvest location and land base restrictions, and the net present values (NPVs) for each scenario are given in Table 2.4. The table shows that all the model runs produced satisfactory results, with most deviations less than 3%. Furthermore, the differences between objective and lagrangian function values, which are a measure of model convergence, were very small in all model runs. The models with the overlapping tenure constraints have lower NPVs than their corresponding models without constraints, which is consistent with our expectations of the effect of these constraints. Comparisons of the Baserun, Scenarios 1, 1a, and 1b, show that the Baserun has the highest NPV as expected. This means that the best tenure policy from an economic point of view is to eliminate all harvest location and land base restrictions. The total loss in NPV of having both harvest area and land base constraints is $171m (difference in NPV between Scenario 1 and Baserun), which represents a loss of about 7% of the objective function value. However, having both constraints in place only marginally increases the cost of the constraints. From Table 2.4, imposing harvest location restrictions alone decreases NPV by $169m. Imposing land base restrictions on top of harvest location restrictions decreases NPV by only $2m. Similarly, imposing land base restrictions alone decreases NPV by $157m, whilst imposing harvest area restrictions on top of the land base restrictions only decreases NPV by $14m.

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[2] The final solution arrived at is optimal but only nearly feasible. The stopping rule was fairly judgmental as the model was typically allowed to run 400 iterations. Usually after 350 iterations, there were negligible changes in both the average deviations and objective function values.

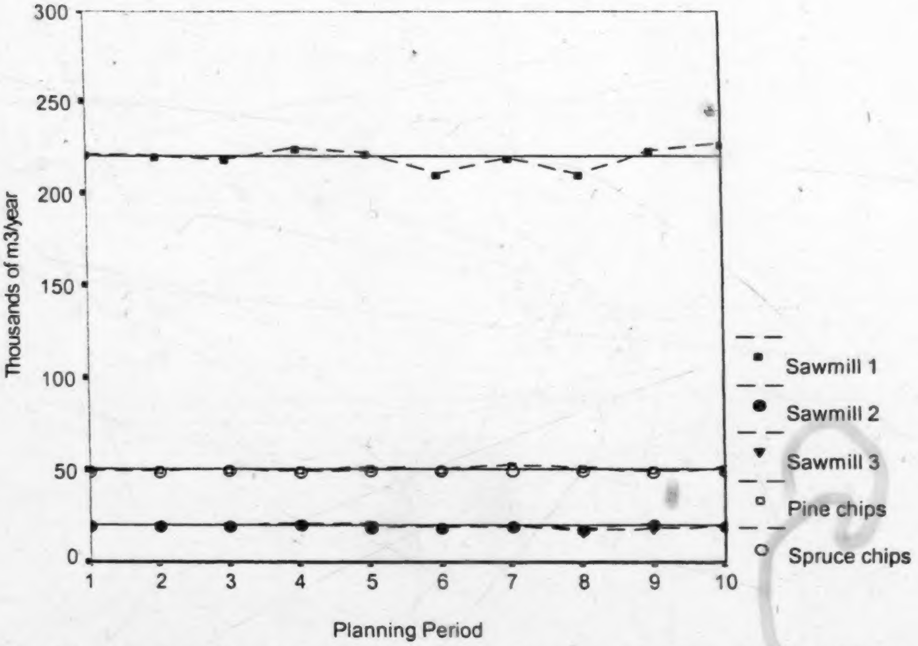

Figure 2.1. Simulated lumber and chip outputs for the sawmills and chip mill for the Baserun.

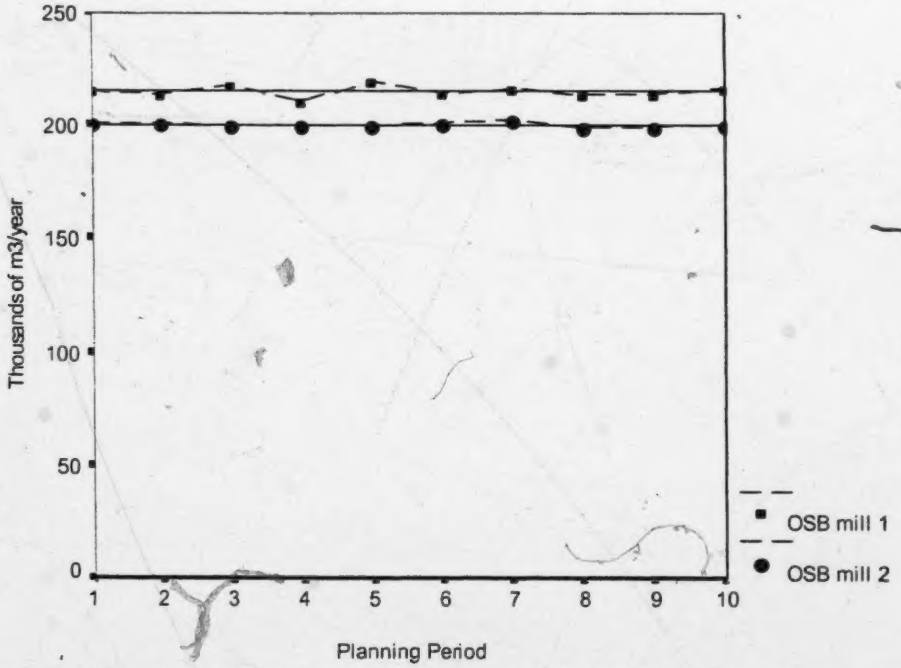

Figure 2.2. Simulated OSB outputs for the two OSB mills for the Baserun.

Table 2.4. Average deviations, number of constraints violating the overlapping tenure constraint, and net present values for the eight scenarios.

Model run	Average absolute deviation (%) from mill demand constraints						LBR and HLR*	Objective function (10^9 \$)	% difference between Obj. fun. and Lag. values
	1	2	3	4	5	6			
Baserun	1.80	2.19	4.64	0.89	0.36	0.94	-	2.311	0.000
Scenario1	4.79	4.29	5.81	1.34	1.08	5.37	29	2.140	0.002
Scenario1a	3.06	3.48	3.97	1.82	0.67	3.14	28	2.142	0.001
Scenario1b	1.70	3.12	5.65	0.37	0.80	0.61	1	2.154	0.001
Scenario 2	3.52	2.03	6.06	0.83	0.30	3.26	30	2.330	0.002
Scenario 3	5.38	4.10	6.23	1.26	0.58	5.43	29	2.302	0.002
Scenario 4	0.66	1.25	2.09	0.87	0.13	0.34	-	2.533	0.000
Scenario 5	1.24	2.12	3.91	0.43	0.23	0.55	-	2.477	0.001

*Note: For scenarios 1, 2 and 3, the overlapping tenure constraints are a combination of harvest location and land base restrictions. Scenario 1a is the HLR only, and Scenario 1b is the LBR only.

Shadow Prices on Mill Demand Constraints

The shadow prices on the mill demand constraints for lumber, OSB and pulp mills for both the Baserun and Scenario 1 are shown in Figures 2.3 – 2.5. The correct interpretation of these shadow prices is that these are the marginal costs of regenerating, harvesting, transporting the wood to the millgate, and milling for each mill. First, we describe several patterns of interest in the shadow price charts given in Figures 2.3-2.5.

1) Shadow prices (marginal costs) for Sawmill 3 are significantly higher when overlapping tenure constraints are imposed than in the Baserun. For Sawmill 2, marginal costs are higher when overlapping tenure constraints are imposed except in periods 7 and 10. However, for Sawmill 1 the marginal costs are higher in the first four periods when overlapping tenure constraints are present, after which the marginal costs under overlapping tenure constraints actually decrease below the marginal costs when there are no overlapping tenure constraints. The overlapping tenure constraints restrict some mills' harvesting to certain locations and land bases. Hence, one expects costs to drop once the constraints are removed. The marginal costs for Sawmill 3 are higher with the constraints because the locations that this mill is allowed to harvest from are outside its woodshed if

no constraints were imposed. These results show that although dropping the constraints result in an overall increase in net returns and reductions in costs, the gains are not evenly spread across mills and over time.

2) Marginal costs for the Baserun tend to increase over the planning horizon. These increases are due to long term scarcities that emerge as more of the wood is harvested off the FMAs. Since the Baserun does not contain any overlapping tenure restrictions, the only reason for increasing marginal costs is scarcity of wood over time. The long-term scarcity of wood given these scenarios is also reflected in how the age class distribution changes over time. Figure 2.6 shows that, the average age of the forest gets younger over time.

3) Another pattern that can be seen in Figure 2.3 is that marginal costs for all sawmills tend to be closer together when overlapping tenure constraints are relaxed than when they are applied. This occurs because the only major difference in costs that can exist in the model once overlapping tenure constraints are removed is in transport costs to the mills. At the margin of each mill's woodshed the value of sending wood to the competing mills will be roughly equivalent. This is just another version of the economic criteria for maximization, which says that at a maximum net present value the marginal returns to each land use will be equalized. Alternatively, under overlapping tenure woodsheds are not determined by economic criteria.

4) Shadow prices for OSB mills for the Baserun remain the same or slightly decrease over the planning horizon (Figure 2.4). This reflects a relative abundance of aspen wood on the two FMAs. Dropping the overlapping tenure constraints results in a decrease in the marginal costs of producing aspen chips.

5) Figure 2.5 shows the shadow prices for softwood chips. The results are similar to those for Sawmill 3 since both Sawmill 3 and the pulp mill are located in Whitecourt and are allowed the same harvest locations. Dropping the overlapping tenure constraints

results in very substantial decreases in the marginal costs of producing both types of chips.

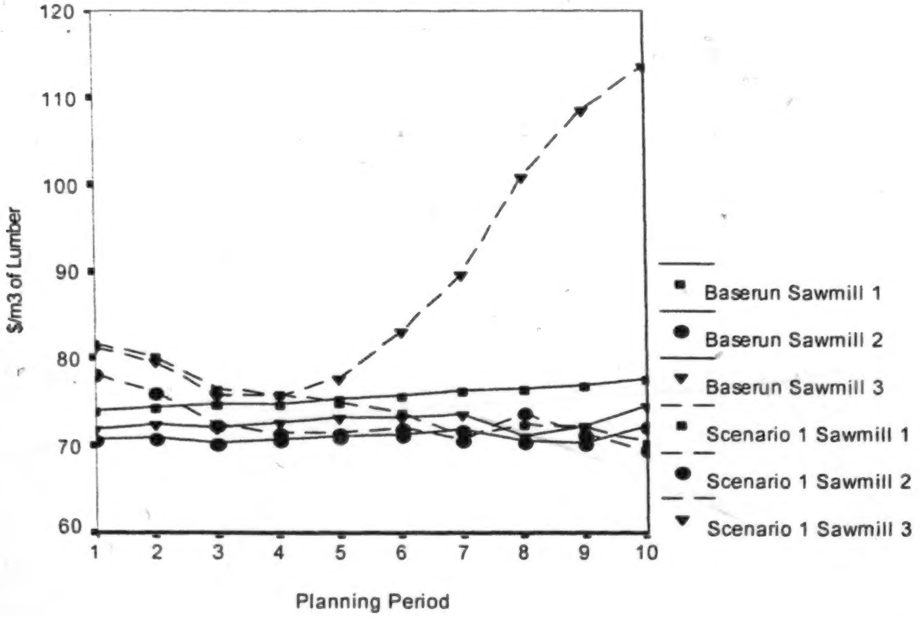

Figure 2.3. Comparison of the shadow prices for lumber mills for the Baserun and Scenario 1

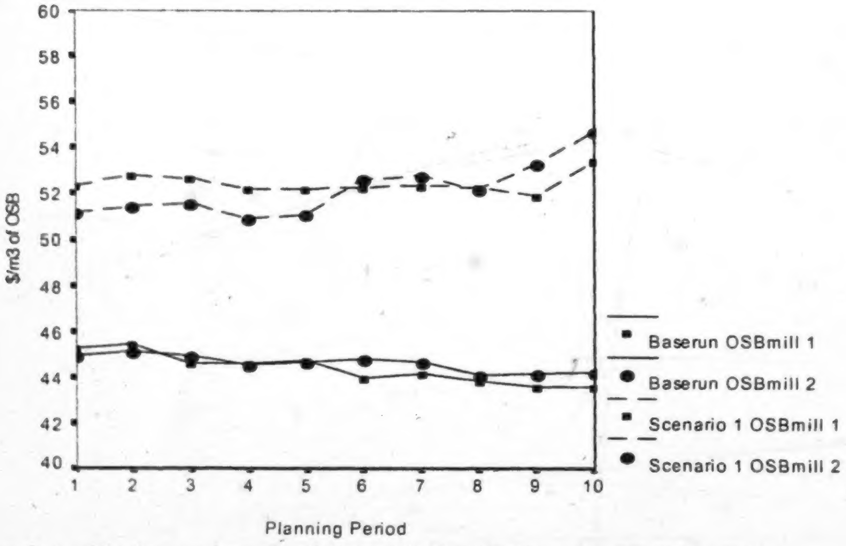

Figure 2.4. Shadow prices for OSB mills for the Baserun and Scenario 1.

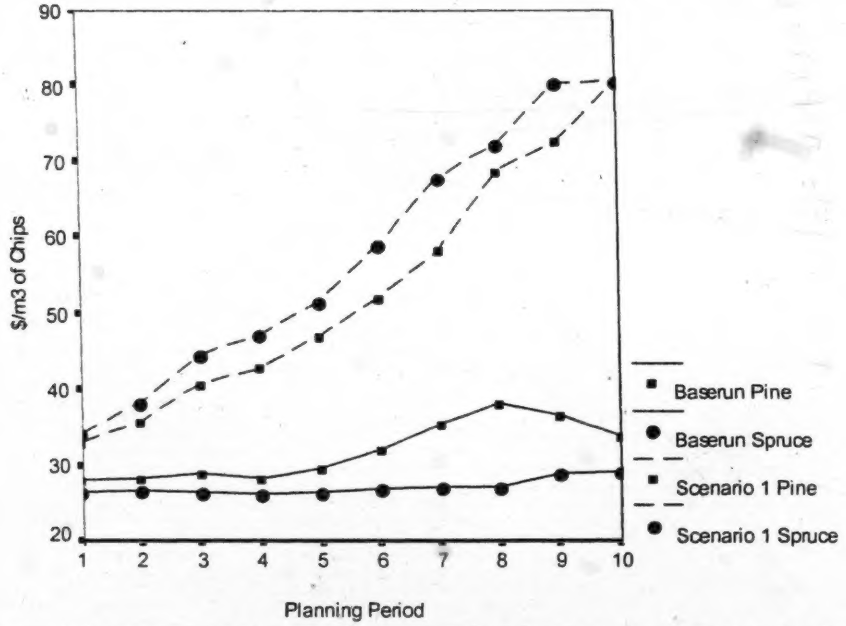

Figure 2.5. Comparison of shadow prices for pine and spruce chips for the Baserun and Scenario 1.

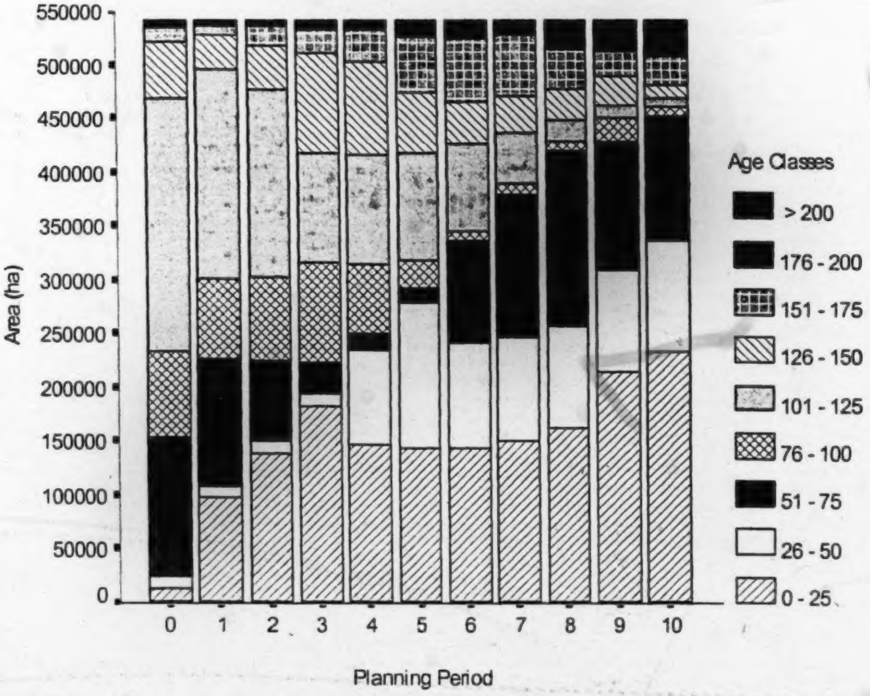

Figure 2.6. Age class distribution for all species at the beginning of the planning horizon and each planning period for the Baserun.

Effect of Overlapping Tenure Constraints on Harvest Schedules

Detailed analyses of the harvest schedules were carried out for the Baserun and Scenario 1. The wood procurement zones for each mill provide insights into why the marginal costs change in the direction they do. The analysis of the harvest schedules for each mill revealed how the transport destinations for wood in each location changes under the two scenarios. The schedules, shown in Figures 2.7 – 2.12, reveal the differences in the supply locations that are harvested and how much is harvested in each supply location. Comparisons of the wood procurement zones for the six mills with and without overlapping tenure restrictions reveal the inefficiency in wood allocation when overlapping tenures are present. In general the wood procurement zones for all mills when harvest area and land base restrictions are removed are closer to each respective mill since without restrictions on where to harvest, mills take wood from supply locations that minimize transportation costs. The boundaries of the allowed locations for each mill are illustrated with dotted lines in the respective maps.

The wood procurement zones for Sawmill 1 are shown in Figure 2.7a and 2.7b. In the Baserun, without overlapping tenure constraints, sawmill 1 takes wood from locations that are closer to the mill compared to when there are restrictions. When there are restrictions, harvests for Sawmill 1 spread over a larger area and in particular they spread into the northern parts of the Edson FMA area. Marginal costs increase for Sawmill 1 in the short run because Sawmills 2 and 3 are forced to harvest in areas where it is more profitable for Sawmill 1 to harvest, thus pushing Sawmill 1's harvest into less profitable areas. However, Sawmill 1's marginal costs eventually decrease when restrictions are imposed because in the long run there is more wood available to Sawmill 1 because Sawmill 2 and 3 are restricted to small areas within the FMA area.

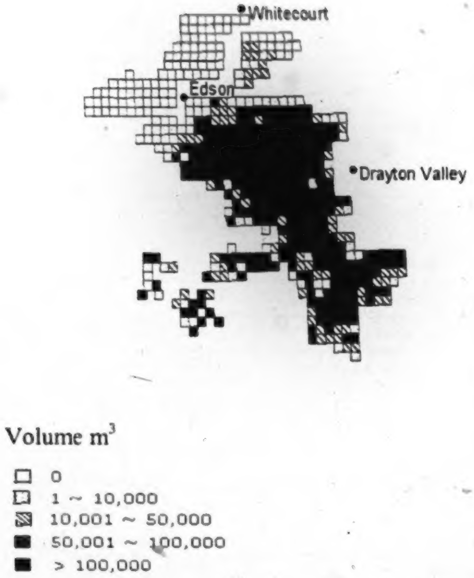
Whitecourt
Edson
Drayton Valley
Volume m³
☐ 0
☐ 1 ~ 10,000
▨ 10,001 ~ 50,000
■ 50,001 ~ 100,000
■ > 100,000

Figure 2.7a. Wood procurement zone for sawmill 1 (DV) for Baserun for the planning horizon.

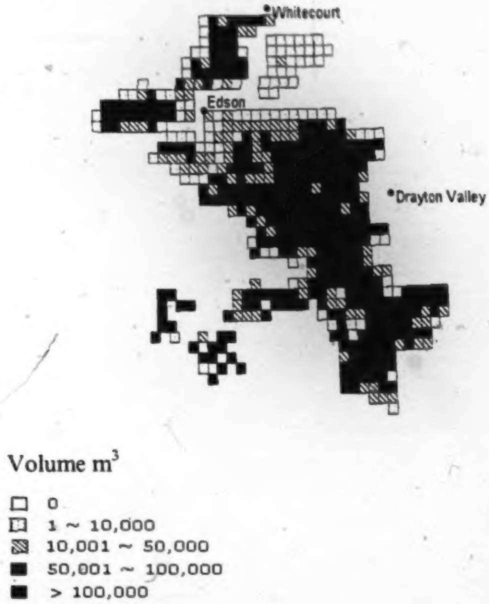
Whitecourt
Edson
Drayton Valley
Volume m³
☐ 0
☐ 1 ~ 10,000
▨ 10,001 ~ 50,000
■ 50,001 ~ 100,000
■ > 100,000

Figure 2.7b. Wood procurement zone for sawmill 1 (DV) for Scenario 1 for the planning horizon.

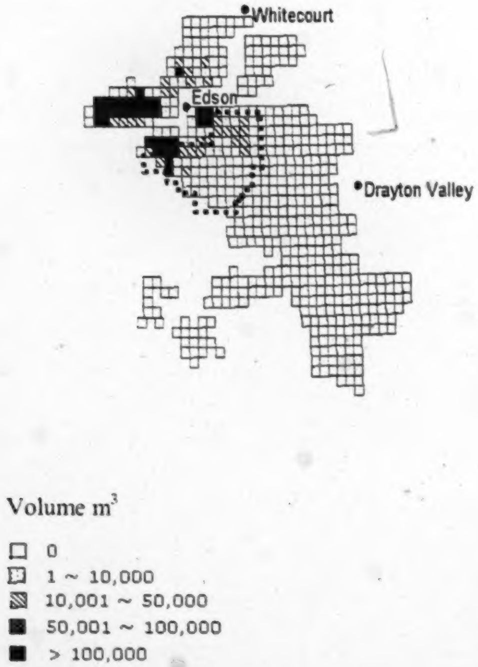
Whitecourt
Edson
Drayton Valley
Volume m³
☐ 0
☐ 1 ~ 10,000
▨ 10,001 ~ 50,000
■ 50,001 ~ 100,000
■ > 100,000

Figure 2.8a. Wood procurement zone for sawmill 2 (ED) for Baserun for the planning horizon. Dotted lines demarcate the allowed harvest locations for the mill.

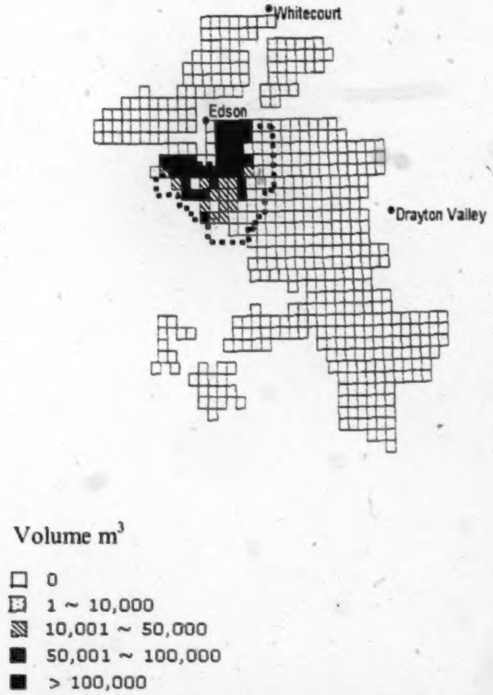
Whitecourt
Edson
Drayton Valley
Volume m³
☐ 0
☐ 1 ~ 10,000
▨ 10,001 ~ 50,000
■ 50,001 ~ 100,000
■ > 100,000

Figure 2.8b. Wood procurement zone for sawmill 2 (ED) for Scenario 1 for the planning horizon. Dotted lines demarcate the allowed harvest locations for the mill.

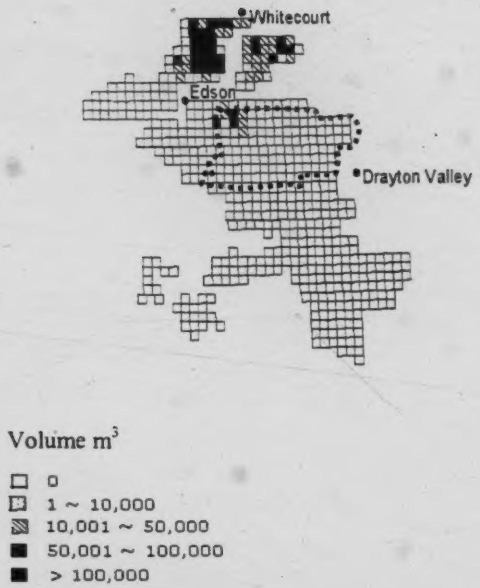
Whitecourt
Edson
Drayton Valley
Volume m3
☐ 0
☐ 1 ~ 10,000
▨ 10,001 ~ 50,000
■ 50,001 ~ 100,000
■ > 100,000

Figure 2.9a. Wood procurement zone for sawmill 3 (WC) for Baserun for the planning horizon. Dotted lines demarcate the allowed harvest locations for the mill.

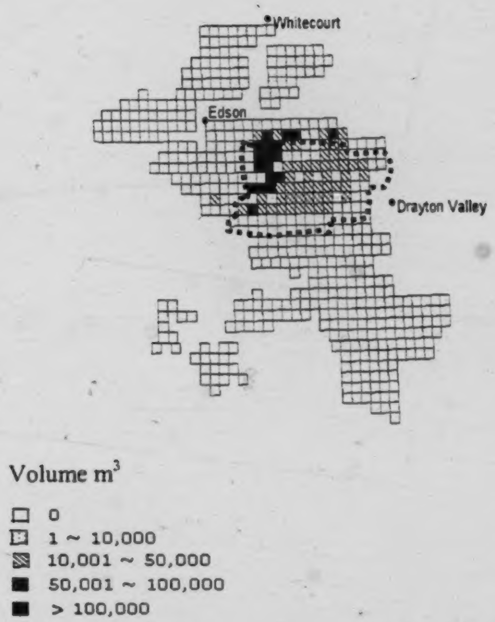
Whitecourt
Edson
Drayton Valley
Volume m3
☐ 0
☐ 1 ~ 10,000
▨ 10,001 ~ 50,000
■ 50,001 ~ 100,000
■ > 100,000

Figure 2.9b. Wood procurement zone for sawmill 3 (WC) for Scenario 1 for the planning horizon. Dotted lines demarcate the allowed harvest locations for the mill.

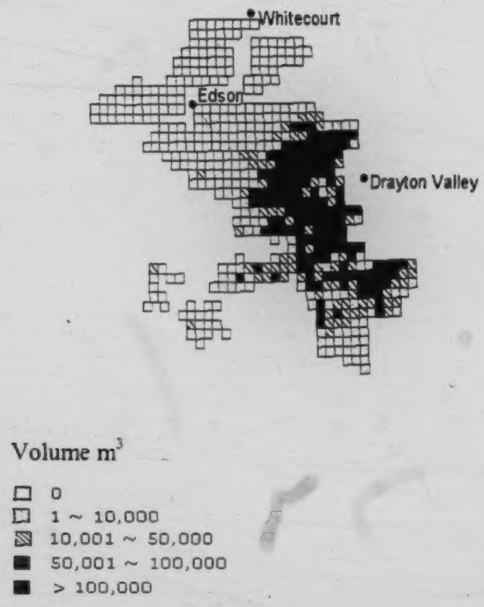
Whitecourt
Edson
Drayton Valley
Volume m3
☐ 0
☐ 1 ~ 10,000
▨ 10,001 ~ 50,000
■ 50,001 ~ 100,000
■ > 100,000

Figure 2.10a. Wood procurement zone for Osb mill 1 (DV) for Baserun for the planning horizon.

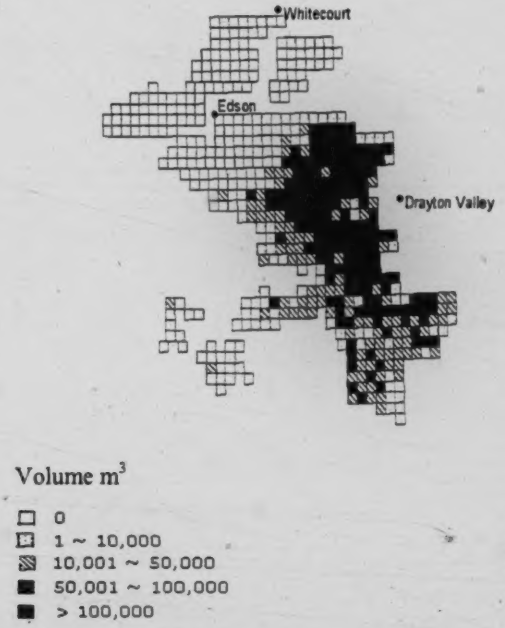
Whitecourt
Edson
Drayton Valley
Volume m3
☐ 0
☐ 1 ~ 10,000
▨ 10,001 ~ 50,000
■ 50,001 ~ 100,000
■ > 100,000

Figure 2.10b. Wood procurement zone for Osb mill 1 (DV) for Scenario 1 for the planning horizon.

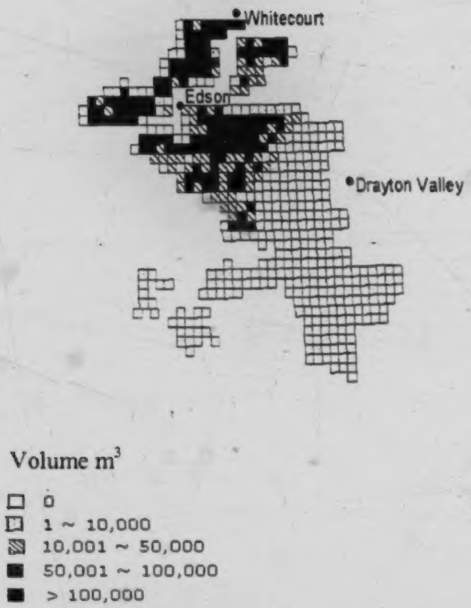
Whitecourt
Edson
Drayton Valley
Volume m³
□ 0
□ 1 ~ 10,000
▨ 10,001 ~ 50,000
■ 50,001 ~ 100,000
■ > 100,000

Figure 2.11a. Wood procurement zone for Osb mill 2 (ED) for Baserun for the planning horizon.

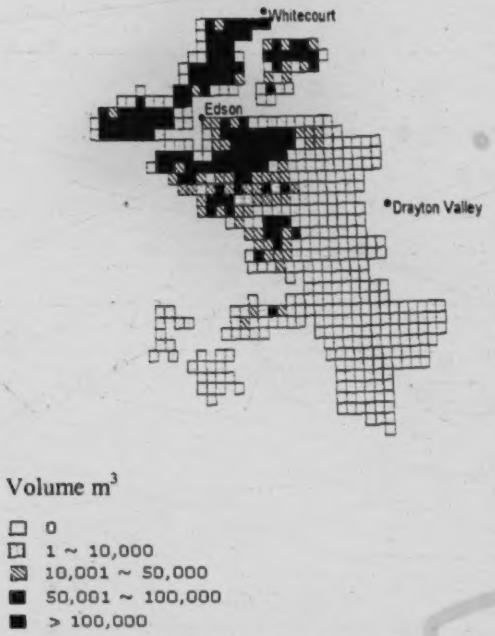
Whitecourt
Edson
Drayton Valley
Volume m³
□ 0
□ 1 ~ 10,000
▨ 10,001 ~ 50,000
■ 50,001 ~ 100,000
■ > 100,000

Figure 2.11b. Wood procurement zone for Osb mill 2 (ED) for Scenario 1 for the planning horizon.

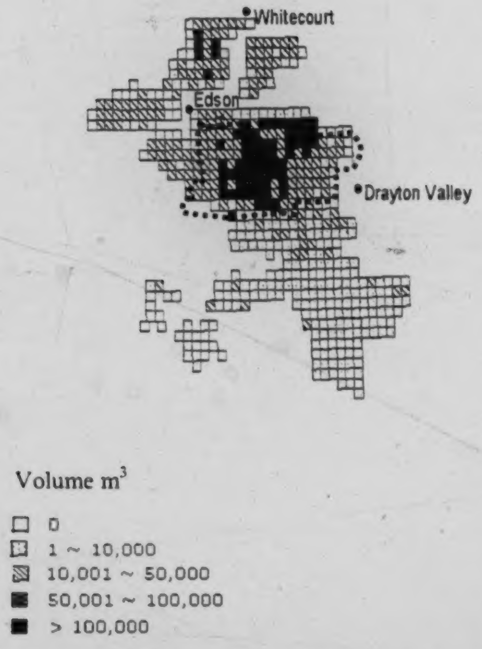
Whitecourt
Edson
Drayton Valley
Volume m³
□ 0
□ 1 ~ 10,000
▨ 10,001 ~ 50,000
■ 50,001 ~ 100,000
■ > 100,000

Figure 2.12a. Wood procurement zone for pulp mill (WC) for Baserun for the planning horizon. Dotted lines demarcate the allowed harvest locations for the mill.

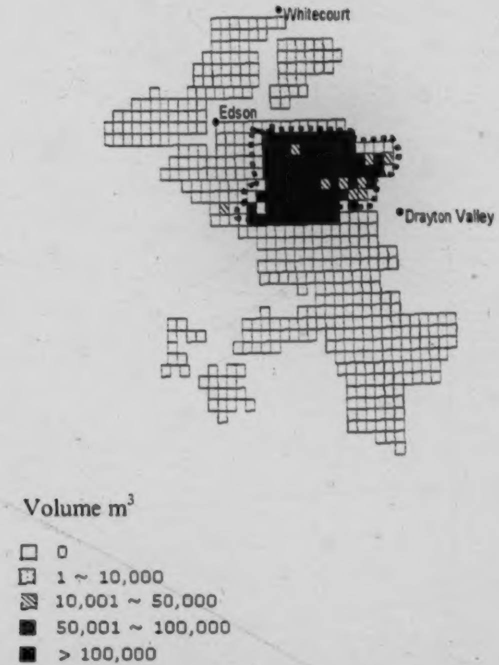
Whitecourt
Edson
Drayton Valley
Volume m³
□ 0
□ 1 ~ 10,000
▨ 10,001 ~ 50,000
■ 50,001 ~ 100,000
■ > 100,000

Figure 2.12b. Wood procurement zone for pulp mill (WC) for Scenario 1 for the planning horizon. Dotted lines demarcate the allowed harvest locations for the mill.

The wood procurement zones for Sawmill 2 are given in Figures 2.8a and 2.8b. The locations that Sawmill 2 has harvesting rights to are shown in Figure 2.8b. Because Sawmill 2 is located close to most of its allowed harvest locations, removal of constraints results in a slight reduction in the marginal costs of producing lumber except in the seventh and last periods (Figure 2.3). Reduction in marginal costs in periods 7 and 10 when the restrictions are imposed suggests that there is more wood available to Sawmill 2 in its allowed harvest locations in those two periods.

Figures 2.9a and 2.9b represent the harvest schedules for Sawmill 3 with and without the overlapping tenure constraints. The allowed area of harvests for this sawmill is shown in Figure 2.9b. Without the constraints, the woodshed for Sawmill 3 is shown in Figure 2.9a. The allowed locations for Sawmill 3 are the farthest from the mill location among all three sawmills. In fact, the woodshed without the constraints is closer to the mill than all locations where it has rights to harvest. Another reason for the high marginal costs when overlapping tenure constraints are imposed may be that Sawmill 3 is forced to harvest from locations with lots of deciduous species or areas that are uneconomic to harvest. Therefore, this results in significant reductions in the marginal costs in all periods when the constraints are removed. It is however, important to point out that these shadow price are only for the part of Sawmill 3's wood supply that comes from the FMAs under study. Sawmill 3 has other wood allocations in other FMAs that are not taken into account in this study.

The schedules for the two OSB mills are given in Figures 2.10a and 2.10b for OSB mill 1 and Figures 2.11a and 2.11b for OSB mill 2 respectively. The FMA holder operates both OSB mills, and there are no restrictions on where any of the two OSB mills harvest their wood. Therefore, wood can flow across the FMA boundaries. Although the OSB mills are not restricted from harvesting from any location, the schedules are seen to differ under the Baserun and Scenario 1. In particular, the presence of overlapping tenure constraints results in spreading out of harvests and consequently, higher marginal costs for OSB (Figure 2.4). It is not particularly obvious why harvests for OSB mills should spread out when the overlapping tenure constraints are imposed. It is likely that there is less wood available to the OSB mills in close by locations because of the land base restrictions, which makes it more difficult for the sawmills to cut in the more mixed stands.

Figures 2.12a and 2.12b represent the harvest schedules for the pulp mill located at Whitecourt with and without the overlapping tenure constraints. The allowed area of harvests for this pulp mill is shown in Figure 2.12b. But without constraints, the woodshed for the pulp mill is shown in Figure 2.12a. Removal of constraints results in a decrease in marginal costs in all planning periods. The woodshed without the constraints shows that wood is harvested from almost every part of the two FMAs to the pulp mill. The reason for the differences in the wood procurement zones appears to be related to wood sorting. There are efficiencies built in the model so that stands can be scheduled to deliver product to multiple mill destinations. When there is no constraint on wood harvesting wood is taken from many places in the management unit because pulpwood components of conifer stands are shipped to the pulp mill and sawlogs are shipped to sawmills. When the constraints are put in place the pulpwood that could have been delivered from the north of Edson (Figure 2.12a) (which would have been efficient) can no longer be delivered under the constraints. All the pulpwood must be delivered from within the narrowly defined supply area of Figure 2.12b.

Marginal Costs of Overlapping Tenure Constraints

Table 2.5 shows the shadow prices or marginal costs of the overlapping tenure constraint set given by Equation 2.6. All three sets of shadow prices are described by the same constraint except that the set over which the summation takes place is different in each case. The first set is a combination of restrictions on harvest area and land bases where mills can take their wood. The second is the harvest area restrictions alone, whilst the third is the land base restrictions. These shadow prices represent the marginal reduction in the objective function value as a result of not allowing a small amount of wood to be harvested from the areas (or land base) from which the mill is restricted from harvesting. Hence, since the shadow price for Edson sawmill is \$3.62/m^3 in the first period the overall objective function would increase by \$3.62 for every m^3 of wood that it could harvest from outside the area and land bases from which it is currently allowed to harvest. The shadow prices for the OSB mills are zero because there are no restrictions on where these mills can harvest. An examination of Table 2.5 reveals that for the combined harvest area and land base restrictions, the shadow prices on Sawmill 3 and the pulp mill (both in Whitecourt) are higher than the other mills. This is because the FMU allocated for harvesting to these two mills is far outside the two mills' woodshed if there were no restrictions. In contrast, Sawmill 1 and 2 are located close to the FMUs that they have harvest rights to (compare Figures 2.8a and 2.8b with Figures 2.9a and 2.9b), and so the marginal costs of the overlapping tenure constraints are lower. The shadow prices on the harvest area restrictions are much larger than those of land base restrictions. This is because the land base restrictions prohibit a mill from harvesting deciduous land base if it is a conifer mill and vice versa – so mills are being prevented from harvesting wood from areas that they are less likely to want to harvest in the first place. This makes the harvest area restrictions more binding than the land base restrictions.

Table 2.5. Final shadow price estimates of the harvest area and land base restrictions for Scenario 1 by period.

Constraint Type	Planning Period									
	1	2	3	4	5	6	7	8	9	10
Harvest Location and Land Base Restrictions										
Sawmill (DV)	3.62	4.73	9.62	9.36	13.15	15.67	21.96	18.69	22.10	22.39
Sawmill (ED)	6.93	6.45	8.56	8.88	13.03	16.12	23.23	20.88	23.01	23.32
Sawmill (WC)	6.58	6.92	10.27	11.16	17.79	22.89	33.92	35.83	43.73	47.88
Pulp Mill (WC)	15.35	13.85	15.37	18.97	20.78	26.94	39.59	37.96	46.32	47.88
Harvest Location Restrictions										
Sawmill (DV)	3.06	4.28	9.41	8.49	13.70	16.73	22.30	23.68	26.62	27.13
Sawmill (ED)	6.93	5.67	9.44	7.99	13.30	16.58	22.09	23.51	27.10	26.09
Sawmill (WC)	6.58	7.20	10.67	10.30	17.16	21.57	29.98	27.10	27.29	32.27
Pulp Mill (WC)	15.35	12.89	15.37	18.14	18.87	24.33	33.38	32.00	38.52	37.15
Land Base Restrictions										
Sawmill (DV)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Sawmill (ED)	6.25	1.55	1.03	6.75	6.72	1.37	2.81	5.53	5.57	0.59
Sawmill (WC)	2.97	1.53	0.00	0.06	1.17	0.16	0.21	0.32	3.42	0.15
Pulp Mill (WC)	7.49	7.34	9.49	11.87	14.15	9.02	11.56	13.23	22.34	10.61

Note: The shadow prices for OSB are zero and not included because there are no restrictions on where they should harvest.

Overlapping Tenure Constraints and Differential Allocation of Increased AAC

In order to put the analysis of overlapping tenure constraints in the context of forest management policy in Alberta, we examine the interrelationships between different allocations of increased allowable cut and the overlapping tenure constraints. The primary objective is to examine how different allocations of increased AAC to the mills affect the overlapping tenure constraints shadow prices. This is achieved using results from Scenarios 2 and 3. In Scenario 2, an increase in the conifer AAC by 20,000m^3/year is allocated to the FMA holder only. In Scenario 3, this additional AAC is distributed to all tenure holders (mills) according to their shares of AAC for the FMA.

The shadow prices of the overlapping tenure constraints for Scenario 2 and 3 are given in Table 2.6. For Sawmill 1, the overlapping tenure shadow prices were higher in Scenario 2 for the first four periods compared to Scenario 3 and then lower for the remaining six periods in the planning horizon. We expect that since the AAC for Sawmill 1 in Scenario 2 is higher than in Scenario 3, all things being equal, the shadow prices of the overlapping tenure constraints should be higher in Scenario 2 than Scenario 3 in all periods. The reason for the decrease in the shadow prices for Sawmill 1 in later periods in Scenario 2 is that in the long run there is more wood available to Sawmill 1 because Sawmill 2 and 3 are restricted into smaller areas on the landscape. For Sawmill 2 and 3 and the pulp mill, the shadow prices of the overlapping tenure constraints in Scenario 3 are consistently higher in Scenario 3 than in Scenario 2, mainly due to the higher AAC in Scenario 3 than Scenario 2. Therefore in general, increasing the AAC increases the shadow prices on the overlapping tenure constraints.

Table 2.6. Final shadow price estimates of the overlapping tenure restrictions for Scenario 2 and Scenario 3.

Constraint Type	Planning Period 1	2	3	4	5	6	7	8	9	10
Scenario 2										
Sawmill (DV)	2.26	7.00	12.63	18.11	22.79	26.30	28.87	33.30	36.50	37.97
Sawmill (ED)	3.70	5.90	11.87	17.74	22.58	26.03	30.60	36.51	37.87	37.84
Sawmill (WC)	4.67	9.07	14.52	20.46	28.43	35.60	45.72	57.71	66.07	78.51
Pulp Mill (WC)	14.12	13.62	16.32	21.99	31.32	40.21	52.36	59.68	74.01	78.58
Scenario 3										
Sawmill (DV)	2.01	6.52	11.04	15.87	22.96	26.31	30.73	38.52	42.61	47.11
Sawmill (ED)	3.88	6.38	8.85	15.84	23.13	28.59	34.60	43.55	45.54	47.51
Sawmill (WC)	4.75	10.80	12.20	20.44	32.25	42.65	57.11	70.64	77.05	85.14
Pulp Mill (WC)	14.38	13.80	17.04	23.62	33.80	45.01	61.08	70.75	78.68	85.29

Effect of Increased AAC on Marginal Costs

Next, we consider the effect on marginal costs of the three final products following an increase in AAC with and without overlapping tenure constraints. The hypothesis is that the increase in marginal costs due to an increase in AAC is higher under overlapping tenure constraints than without the constraints. Table 2.7 presents the average 10-year differences in marginal costs with and without overlapping tenure constraints for all six mills.

Table 2.7 Average 10-year differences in marginal costs ($/m^3) between selected scenarios.

Differences	Sawmill 1	Sawmill 2	Sawmill 3	Osb mill 1	Osb mill 2	Pine chips	Spruce chips
Scenario 1 minus Baserun	-0.736	0.43	15.87	8.05	7.56	21.16	30.37
Scenario 3 minus Scenario 5	-3.96	1.71	33.61	7.90	7.61	38.34	45.57
Scenario 3 minus Scenario 1	1.32	1.62	18.23	0.11	0.35	20.94	15.90
Scenario 5 minus Baserun	0.57	0.57	0.05	0.03	0.24	3.69	0.55

A comparison of the difference in marginal timber costs in Scenario 3 and 5 to the difference in marginal costs for Scenario 1 and the Baserun, shows that overlapping tenure constraints have a greater effect when allowable cut levels are higher. With the exception of Sawmill 1 and OSB mill 1 the marginal costs of the demand constraints all increased by a greater degree when overlapping tenure constraints were present (compare rows 1 and 2 in Table 2.7). Another way of looking at the costs of the overlapping tenure constraints is to examine the change in marginal costs of production when allowable cut is increased with and without the constraints. Results in rows 3 and 4 of Table 2.7 show that when allowable cut is increased the marginal cost increases are much lower when there are no overlapping tenure constraints present. Increasing cut levels is less expensive when the constraints are not present.

DISCUSSION

The approach presented here as well as the results have several management applications. The application of the model in this paper shows the method is effective for generating optimal near feasible solutions within a short computer runtime of about 30 min. In the context of the scenarios examined in this paper, the model supplies data on the marginal cost of wood and the marginal cost of overlapping tenure constraints. The model shows how inefficiencies in wood allocation, in this case imposed by overlapping tenure constraints, can affect the costs of supplied wood to different mills over time and space. As revealed in the different scenarios, the model is also capable of showing that gains (or losses) from policy changes can be uneven. The allocation of wood under the scenarios without restrictions is efficient while the allocation under the scenarios with constraints is not. The gains and losses depend on how far each mill is from the locations that it has rights to harvest from. If a mill is located far away from its allowed locations, (outside of the locations from which it would have harvested wood without any constraints) then removal of the constraints results in gains to that mill. On the other hand, if a mill is located within its economic woodshed, then removal of constraints may result in losses. In the scenarios examined, Sawmill 3 and the pulp mill, both of which are located in Whitecourt, and far away from their allowed harvest locations, gained from the removal of the overlapping tenure constraints. The marginal costs for the other mills either remained the same or decreased only slightly when the constraints were removed. Therefore, the most efficient wood allocation may be achieved by eliminating all overlapping tenure constraints.

A detailed analyses of the harvest schedules revealed that under overlapping tenures, wood to some mills have to be transported on distances greater than when the restrictions are removed. The increased marginal costs in the overlapping tenure runs could be interpreted in a couple of ways. First, the increased marginal costs imply that the marginal value of timber (marginal value = marginal cost) increases in the future, given that the assumptions about the demand in the future are correct. This suggests that prices may become high enough to justify more intensive siliviculture. The second and alternative interpretation is that the demand scenario is incorrect and that the implied marginal value or price of timber is too high. In this case the model's demand specification should be reformulated to allow wood harvests to decrease thus decreasing wood harvests in the future. Hence, the model provides a way of tying the wood production to marginal costs and values of timber, which can be compared to expectations of future timber prices. This provides valuable information for supply planning and current planning in silvicultural investment expenditures.

One of the major concerns about overlapping tenures is the incentive for firms to invest in intensive silviculture. The results from the model runs show that for all the scenarios investigated, investments in basic or intensive silviculture were not economically viable options. This suggests that the final prices of wood products from our models are not high enough to justify any investment in silviculture. These results made it difficult to look at incentives created by the allowable cut constraints. Sensitivity analysis together with further runs could be used to investigate when investment in intensive silviculture becomes profitable.

Although the scenarios examined here did not include non-timber values there is no reason that non-timber values could not be included. This would require the model to track attributes of the forest that are linked to non-timber values. Constraints on the levels of non-timber value attributes could be added and shadow prices computed for these constraints in a similar manner as the timber value constraints described in the current paper. Alternatively, in some cases, non-timber values could be incorporated directly into the objective function. For example, recreational forest user utility functions derived from Random Utility Models could be incorporated into this framework. Both of these approaches could be extremely useful for evaluating landscape management strategies such as TRIAD (Seymour and Hunter, 1992). In the models reported here each analysis area was included into a supply location defined as a one-quarter township (5 x 5 km). Making the size of the spatial unit smaller might actually improve how the model converges. Because the model described in this paper dealt only with timber supply in the context of overlapping tenure constraints, the model was modified to consider access road costs in the next chapter, and non-timber values in the fourth chapter.

CONCLUSIONS

This study applied an optimization approach to estimate the cost of overlapping tenure constraint on forest management agreement areas in northern Alberta. The results from the various scenarios investigated showed that overlapping tenure constraints are costly. The costs to individual tenure holders are highly dependent on how far the mills are from their allowed harvest locations and how far the constraints shift mill harvest areas away from their optimal wood procurement zones. Although in general removal of the constraints lead to decreased costs, the benefits of removing constraints are unevenly distributed among tenure holders. Removal of the constraints leads to a 7% increase in the net present value of the forest. For mills that are located within short distances of their allowed harvest locations, removal of constraints do not significantly lower marginal costs. The results also showed that increases in AAC are more costly when overlapping tenure constraints are present than when they are not present.

While the results here suggest that overlapping tenure constraints should be removed and better ways of allocating land for harvest should be sought, they also suggest that in some cases, removal of overlapping tenure constraints may decrease flexibility for some mills resulting in increases in costs for those mills when overlapping tenure constraints are removed. This suggests that although policies to remove overlapping tenure constraints would be efficient, they will be opposed by some tenure holders that derive an economic advantage from existing arrangements. In these cases some means of compensating mills that lose as a result of more efficient wood allocation may have to be arranged.

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CHAPTER 3
A DECOMPOSITION APPROACH TO INTEGRATED FOREST HARVEST SCHEDULING AND ACCESS PLANNING

INTRODUCTION

The construction of access roads and the determination of efficient access paths within a forest are central issues in long-term forest management planning for several reasons. First, due to the substantial capital investments required in roads, a large fraction of total forest management costs is spent on road construction or upgrading. Secondly, adequate access must be provided before management activities can be carried out in forests (Weintraub and Navon, 1976). Incorporating access in strategic forest planning models is also important because of the cumulative effects of access provision on non-timber values in the forest. For example, the level of access within a forest may affect the welfare of hunters and recreationists (e. g., McLeod *et al.*, 1993; Adamowicz *et al.*, 1997). Consequently, forest managers and researchers are interested in formulating and developing solution techniques for forest management planning problems that incorporate access.

A wide variety of optimizing models in forest planning have been used to analyze separately silvicultural and transportation problems (Bare, 1972). In this method, road building and timber management are planned separately, that is, road planning and/or construction usually precede harvest planning. Stands are thereafter scheduled for harvesting based on their accessibility. Weintraub and Navon (1976), however, argue that this sequential nonintegrated approach can lead to suboptimization on two counts. First, the wrong set of stands may be made accessible; and secondly, the choice of the period of access to each stand may not be optimal. Based on these two reasons, discounted costs of construction, maintenance and hauling may be higher than absolutely necessary, and the impossibility of carrying out silvicultural treatments at the appropriate time may reduce gross timber revenues. When access road construction plays an important role, a more accurate way of dealing with this problem is to represent explicitly in the same model both access and timber resources management activities. Weintraub and Navon (1976) and other earlier studies that integrated road network into timber scheduling e. g., Kirby (1973), Barnes and Sullivan (1980), Sullivan (1973) used mixed integer programming (MIP) for planning the development and use of transportation network. Simultaneous optimization of forest management activities and access using MIP is easy to solve for small models. However, the difficulty of solving such models increases as the number of decision variables and constraints increase. Although these previous problems were of practical sizes for solutions on computers they were not realistic in a practical sense given the reality of forest management on the ground. More recent empirical work that used MIP has relied on random search or heuristic approaches because of the problems associated with large size in anything but small MIP formulations. Most of these approaches do not lend themselves to intertemporally optimized harvest schedules nor do they provide the shadow price information on any of the constraints incorporated in the formulation. Furthermore, most of these models focus on logging access at the operational level, rather than the strategic level of planning.

Our paper addresses these shortcomings by developing a mixed-integer non-linear programming (MINLP) model that integrates access road development with forest harvest scheduling. We apply the model to an operationally sized timber management problem in Alberta, Canada. The main objectives were to: 1) develop a model that integrates access planning into a large, spatially detailed strategic forest scheduling model, and 2) examine the effects of explicitly including access development costs on the harvests schedule and road development. The solution technique of this model is based on an extension of the dual decomposition approach introduced by Hoganson and Rose (1984). The extension incorporates mixed integer programming and uses the theory of lagrangian relaxation discussed by Geoffrion (1974) and Fisher (1981, 1985). While the timber supply model is formulated for a real land base on Weyerhaeuser Forest Management Agreement (FMA) area in Drayton Valley, Alberta, we have modified mill demands slightly. The forest management model described here incorporates multiple products, multiple supply locations, and silvicultural investments in forestry and access road construction. Access to timber supply locations is represented by 0-1 variables, leading to a mixed integer problem. The resulting formulation allows for alternative access directions to each location and the intertemporal optimization of road access decisions. The number of integer decision variables in this formulation is estimated at 3850, which in a branch and bound algorithm would have 2^{3850} combinations of solutions although the number of combinations that actually need to be explored is somewhat less than this because of the constraints on sequencing. For example, some locations would have to be accessed through other locations. Even then, it is obvious that the enormous number of feasible solutions associated with this problem will make it an extremely difficult problem to solve using the branch and bound technique. The solution algorithm we employ breaks down the large MINLP problem into simple stand level economic analysis over the planning horizon, and simple optimal path network analysis for access planning in each planning period given the value of current dual prices. Simple intuitive price adjustment procedures are used to change dual prices to move the solution towards feasibility.

The application of this approach to modeling harvest scheduling and access in this paper is unique in a number of ways. First, it attempts to optimize forest management activities and road access decisions simultaneously, while satisfying multiple mill demands. As far as we know, most previous studies that have solved the access problem either with MIP or using heuristics have not incorporated the important fact that wood will be transported on these access roads to satisfy mill demands in different locations and with different products. The solution to our model is optimal but near feasible in that mill demand constraints are allowed to deviate from timber supply in each period by at most 3%. Secondly, the model converged within a reasonably short period of computer time (20 minutes on a microcomputer with a Pentium III 500 Mhz microprocessor), which is remarkable, given the large size of the model. The flexibility inherent in our modeling approach allows the path of minimum cost to be determined endogenously within the level of detail that road access is modeled. Finally, the approach, which is consistent with economic theory, allows us to estimate the shadow prices on all demand and access constraints. These shadow prices provide useful information about the costs of production or future timber prices. The shadow prices on the demand constraints represent the marginal costs of producing the outputs from the demand centers, whilst the shadow prices on access constraints indicate the average road construction costs per cubic meter of timber harvested from the stands over time. The shadow price information is valuable for supply planning and current planning in silvicultural and access road investment expenditures.

The rest of the paper is organized as follows. The next section gives a brief background to previous forest harvest scheduling studies incorporating access. This is followed by a detailed description of the forest management problem, the non-linear programming formulation, and the solution method. The results from an empirical application of the model is then presented, followed by a discussion of the management applications of the results, and possible extensions of the present model. The conclusions from the results are presented in the last section.

LITERATURE REVIEW

Kirby (1973) was probably the first to recognize the advantages of jointly modeling forestry activities with their required road network. Kirby's paper offered the stimulus in the US Forest Service to re-examine the long-standing sequential planning of roads and management activities. The model was a mixed integer program, which was formulated to maximize benefit from forestland less costs (including road construction costs), subject to management constraints, and relations between adjacent roads.

To account for both road construction and transportation costs (transportation cost was not included in the model by Kirby (1973)), Barnes and Sullivan (1980) and Sullivan (1973) developed a MIP model for forest network planning to maximize net revenue (timber revenue net of road construction and transportation costs). In order to keep the MIP problem to a manageable size, Sullivan (1980) generated *K*-shortest paths between each timber sale and demand points. Optimization was therefore restricted to these paths, which was a small select number of all possible paths.

Weintraub and Navon (1976) combined the network analysis scheme of Sullivan (1973) and proposed an integrated approach, which was applied to a hypothetical forest, covering an area of 100,000 acres. The resulting mixed integer-programming model had 118 timber management activities, with 215 constraints and 256 variables, 24 of which were 0-1 integer variables corresponding to building road segments. The results from the example showed that by jointly considering transportation and timber management, an increase in discounted net revenue of 7% was achieved, 6% of which was from savings in road building and fixed maintenance costs. Their model considered only one demand location and no differentiation of forest products. Also, both the area and the number of activity variables and constraints were relatively small. The network was also simplified by approximating the road network by a hypothetical one consisting of only major corridors.

It was realized in practice that for large and complex networks, optimization based on paths consumes much analyst's time in manually adding paths and may lead to poor solutions as well (Kirby *et al.*, 1986). This is a result of the fact that road construction costs are not used in generating shortest paths. There was no simple way of incorporating road construction costs for each path since many other paths may share a portion of this cost. Kirby *et al.* (1979) overcame this difficulty by considering all paths using the classical transshipment formulation.

Whilst the formulations of mixed integer problems that address stand level and access decisions are not difficult in principle, the main disadvantage is that these problems cannot be solved in a reasonable amount of time because of the integer restrictions. To overcome this, more recent studies have relied on heuristic-based approaches. One of the early applications of such heuristic-based methods is that by Bullard *et al.* (1985), who modeled forest scheduling using random search algorithms. O'Hara *et al.* (1989) also developed randomized search heuristics, which pre-biased the selection of stands based on volume and adjacency. However, because this study used a volume maximization objective function, transportation and road construction costs were not considered, although the model performed well compared to the optimal LP solution.

In order to correctly represent the costs of road and transportation, Nelson and Brodie (1990) developed a random search heuristic that maximized present value minus road construction costs in the objective function. Other studies that used simple heuristics are those of Clements *et al.* (1990), Walters (1991) and Nelson and Finn (1991). Another heuristics approach that has been used to solve large forest scheduling problems is the simulated annealing algorithm that was presented by Kirkpatrick (1983). This method has also been applied by Lockwood and Moore (1993), using penalty costs to ensure that volume and adjacency constraints are met. This study and others by Dahlin and Sallnas (1993) showed that the simulated annealing approach is a viable method to solving large problems. Murray and Church (1995) compare the simulated annealing method to the Tabu search and pairwise interchange approaches and concluded that the Tabu search performed best. A more recent three-stage heuristic for solving harvest scheduling with access road network is reported by Clark *et al.* (2000). The procedure was shown to work well within a very short computer time; however, the application was for a small forest of about 3600 ha. The performance of the model for large problems remains untested and no particular demand locations were identified and included in the model.

METHODS

Model Formulation

To illustrate the usefulness of our modeling approach in providing meaningful information in a realistic setting, a mixed integer non-linear programming formulation was used to develop the timber management schedule for the Weyerhaeuser Forest Management Agreement (FMA) area in Drayton Valley, Alberta. The model is an extension of the Model II structure formalized by Johnson and Scheurman (1977). The problem being addressed in this study can be described as using optimization techniques to formulate and solve an integrated forest scheduling and access road construction activities. Using spatial data from geographic information system, the forest area was aggregated into 577 locations, each location being one-ninth of a township (one township ≈ 10,000 ha or 100 km^2). The problem therefore, is to maximize the present value of wood scheduled on these locations less harvest, transportation, regeneration, and road building costs. With regards to road construction, we define locations that currently have a major road through them as permanently opened/accessed locations (POLs). These POLs are assumed to be built and maintained in good condition throughout the planning horizon at some fixed cost. Temporary or intermittent roads are built to each location in each planning period as needed. To reduce the problem to a reasonable level of complexity, we did not explicitly model temporary roads built to access stands within locations. We find the cost-minimizing road network to connect all locations targeted for harvests to the existing roads. Locations to be harvested are determined based on economic criteria of timber benefits of accessing a location exceeding the costs of doing so. We originally considered a formulation similar to Sullivan's (1973) *K*-shortest paths method which restricted optimization to the few selected paths but abandoned it in favor of a more flexible approach. Although the model does not identify the exact location of roads, it however suggests access directions and access timings along corridors in the range of 1/9 of a township. This could, however, be extended to a smaller spatial unit if necessary. The present formulation allows for differential road construction costs to be easily incorporated, although we did not consider this here. Also, at the moment the optimal paths are decided purely on the basis of timber value, but non-timber values could also be included as shown in Chapter 4 of this thesis.

The mixed integer, non-linear programming formulation for this timber supply problem is given by Equations 3.1-3.11. First, we define the following sets and variables: Let

J be the set of all *supply locations* in the forest with j serving as a counter (j=1,…,J).

J^P be the set of permanently accessed supply locations.

J^1 be the set of all permanently accessed locations that are adjacent to at least one location that is not accessed.

I be the set of all *demand locations* with i serving as a counter (i=1,…,I)

I_j^A be the set of all *supply locations* adjacent to j from which product may be shipped. This set is empty for all $j \in J^p - J^1$. (that is, permanently opened locations that are not adjacent to any unaccessed locations).

I_j^B be the set of all *supply locations* adjacent to j to which product may be shipped.

I_j^C be the set of all *demand locations* to which product may be shipped from j. This set is empty for all $j \notin J^p$.

y_{jit}^p be the volume shipped from supply location j to demand location i for $j \in J^p$.

y_{jkt}^A be the volume shipped from supply location j to supply location k. for $j, k \notin J^p$ and $j \in J^1$.

y_{jt}^s be the volume supply at location j.

y_{it}^d be the volume demand at demand location i.

z_{jt} be the 0,1 access variable for location j.

The objective function for the forest management problem is given by Equation 3.1.

$$\max \sum_i \sum_t R(y_{it}^d) + \sum_t \sum_j E_{jt} w_{jt} - \sum_{s=-M}^{T-z} \sum_{s \geq s+z}^{T} \sum_j^{J} c_{stj} x_{stj} - \sum_t \sum_{j \notin J^p} c_{jt}^A z_{jt} - \sum_t \sum_{j \notin J^p} \sum_{k \in I_j^A} c_{kjt}^s y_{kjt}^A$$

$$- \sum_t \sum_{j \in J^p} \sum_i c_{jit}^s y_{jit}^p \tag{3.1}$$

subject to:

$$y_{jt}^s \leq \sum_{s=-T^0}^{T-z} v_{sjt} x_{sjt} + \sum_{k \in I_j^A} y_{kjt}^A \qquad \forall j, t \; (I_j^A = \phi \text{ for } j \in J^p - J^1) \tag{3.2}$$

$$y_{it}^d \leq \sum_{i \in I_j^c} y_{jit}^p \qquad \forall i, t \tag{3.3}$$

$$\sum_{k \in I_j^B} y_{jkt}^A \leq y_{jt}^s \qquad \forall j \notin J^p \tag{3.4}$$

$$\sum_{i \in I_j^c} y_{jit}^p \leq y_{it}^s \qquad \forall j \in J^p \tag{3.5}$$

$$y_{jt}^s \leq y_{jt}^s z_{jt} \qquad \forall j \notin J^p \tag{3.6}$$

$$\sum_{s=-T^0}^{t-z} x_{sjt} = \sum_{k=t+z}^{T} x_{tkj} + w_{jt} \qquad \forall j, t = 0, \ldots, T \tag{3.7}$$

$$\sum_{t=s+z}^{T} x_{sjt} + w_{jt} = A_{sj} \qquad \forall j, s = -T^0, \ldots, 0 \tag{3.8}$$

$$x_{sjt} \geq 0 \qquad \forall sjt \tag{3.9}$$

$$w_{sj} \geq 0 \qquad \forall sj \tag{3.10}$$

$$y_{it}^d \geq 0 \qquad \forall t \tag{3.11}$$

The remaining variables used are defined as:

$R(y_{it}^d)$ = the revenue for wood products at demand center i in period t.

E_{sj} = the discounted value per unit area of managing stand type j starting in period s and leaving the stand type as ending inventory

w_{jt} = area managed of stand type j in period t and left as ending inventory

x_{sjt} = area managed on stand type j in period s and final harvest in period t

The parameters are defined as:

c_{jt}^A = discounted cost of accessing (road construction) location j in period t

c_{kjt}^s = discounted cost/m^3 of shipping wood from location k to j in period t

c_{jit}^s = discounted cost/m^3 of shipping wood from permanently accessed location j to demand center i in period t

A_{sj} = the number of area unit of stand type j in the first period that were regenerated in period s.

c_{sjt} = the discounted cost per unit area of managing stand type j starting in period s and final harvest in period t

v_{sjt} = the merchantable volume per unit in period t, when stand type j is regenerated in period s.

z = minimum time between regeneration and harvest

T = the number of planning periods in the planning horizon

T^0 = number of periods before period zero in which the oldest age class present in period one was regenerated

The first equation of the model, Equation (3.1), is the objective function, which maximises the net present value of the forest. This is represented as the discounted revenue from the sale of final wood products plus the value of ending inventory minus the cost of regeneration and harvesting, road construction costs, costs of shipping wood from one location to an adjacent location, and the cost of shipping wood from a permanently accessed location to the demand center (mills).

Equations 3.2 and 3.3 describe the timber supply and demand system. Equation set (3.2) says that the volume supply of wood in a given location *j* at time *t* is the sum of wood supplied from that location plus any wood that is shipped through that location. The second term on the right hand side of equation 3.2 is only relevant for areas that are not permanently accessed and for areas that are permanently accessed but immediately adjacent to areas that are not accessed permanently. Equation (3.3) is quite straightforward and imposes bounds on the timber flow to mills. These ensure that the wood shipped from permanently accessed locations to the mills is not less than the mill demands. This constraint is only relevant to permanently accessed locations because wood cannot flow from non-permanently accessed locations directly to the mill. All wood flow to the mills has to go through permanently accessed locations.

Equations 3.4 to 3.6 define the access and wood transport from one supply location to another supply location and from supply locations to mills. Equation 3.4 is only relevant for areas that are not permanently accessed. Equation (3.4) is an accounting equation that measures the volume of wood supply in locations. Specifically, it states that the volume of wood shipped from one location to another location cannot be greater than the volume supply of wood in the initial location. Equation (3.5) implies that the volume of wood shipped from a permanently accessed location to a mill cannot be greater than the volume supply of wood in the supply location. Equations 3.4 and 3.5 taken together suggest that wood flows from one location to another in the locations that are not permanently accessed, whilst in permanently accessed locations, wood is shipped directly from supply locations to the mills. Equation (3.6) constrains the model to ensure that each location without permanent access is accessible when it is to be harvested. That is, wood cannot flow from an unaccessed location, which implies access must first be provided before any location can be treated or harvested. It is important to notice that Equation (3.6) is quite different from the rest in that it is a non-linear constraint in which the access variable (z_{jt}) is a binary integer variable.

Equations 3.7 and 3.8 describe the forestland constraints including the initial age class distribution and the dynamics of transition from harvest to regenerated stands. These two equations are part of the standard Model II set-up of Johnson and Scheurman (1977). Equation (3.7) accounts for area regenerated during the planning period. Total area harvested during the planning period plus area left as ending inventory at the end of the planning period should equal area regenerated during the planning period. This constraint ensures that all harvested areas are regenerated. Equation (3.8) defines the total area availability for the forest area regenerated before the planning period (existing stands). Total area harvested during the planning horizon plus area left as ending inventory (at the end of the planning horizon) should equal the initial area (regenerated in period s before planning period).

We now specify the lagrangian function together with the dual variables for each constraint. These dual variables are important for two main reasons. First, the forest-scheduling problem as presented above will be difficult to solve using traditional mixed-integer non-linear programming solution techniques due to its large size (about 2.6 million decision variables and 96 thousand constraints). The simulation approach of Hoganson and Rose (1984), which is used to solve this model, relies on a direct interpretation of the dual problem formed using Equations (3.1) to (3.8). Secondly, some additional insights and relationships to other literature are apparent from analysis of the dual, for example the Faustmann (1849) optimal forest rotation model.

The lagrangian to the primal problem can be stated as:

$$L = \sum_i \sum_t R(y_{it}^d) + \sum_t \sum_j E_{jt} w_{jt} - \sum_{s=-T^0}^{T-z} \sum_{s \geq s+z}^{T} \sum_j^{J} c_{stj} x_{stj} - \sum_t \sum_{j \notin J^P} c_{jt}^A z_{jt} - \sum_t \sum_{j \notin J^P} \sum_{k \in I_j^A} c_{kjt}^s y_{kjt}^A$$

$$- \sum_t \sum_{j \in J^P} \sum_i c_{jit}^s y_{jit}^p + \sum_j \sum_t u_{jt} \left(\sum_{s=-T^0}^{T-z} v_{sjt} x_{sjt} + \sum_{k \in I_j^A} y_{kjt}^A - y_{jt}^s \right) + \sum_{j \notin J^P} \sum_t v_{jt} \left(y_{jt}^s - \sum_{k \in I_j^B} y_{kjt}^A \right)$$

$$+ \sum_{j \in J^P} \sum_t \theta_{jt} \left(y_{it}^s - \sum_{i \in I_j^c} y_{jit}^p \right) + \sum_{j \in J^P} \sum_t \lambda_{jt} \left(y_{jt}^s z_{jt} - y_{jt}^s \right) + \sum_j \sum_{t=0}^{T} s_{jt} \left(\sum_{s=-T^0}^{t-z} x_{sjt} - \sum_{k=t+m}^{T} x_{tkj} + w_{jt} \right)$$

$$+ \sum_j \sum_{s=-T^0}^{0} a_{sj} \left(A_{sj} - \sum_{t=s+z}^{T} x_{sjt} + w_{jt} \right) + \sum_j \sum_t \pi_{it} \left(\sum_{i \in I_j^c} y_{jit}^p - y_{it}^d \right) \qquad (3.12)$$

The first order conditions from Equation 3.12 for the continuous variables are:

$L_{y_{it}^d} = R'(y_{it}^d) - \pi_{it} = 0$

$L_{x_{sjt}} = -c_{sjt} + u_{jt} v_{sjt} - a_{sj} + s_{jt} \leq 0, \quad = 0 \; if \; x_{sjt} > 0 \quad \forall \, j, s = -T^0, ..., 0 \; and \; t = 0, ... T$

$L_{x_{sjt}} = -c_{sjt} + u_{jt} v_{sjt} - s_{sj} + s_{jt} \leq 0, \quad = 0 \; if \; x_{sjt} > 0 \quad \forall \, j, s = 0, ..., T \; and \; t = s + z, ... T$

$L_{y_{kjt}^A} = -c_{kjt}^s + u_{jt} - v_{kt} \leq 0, \quad = 0 \; if \; y_{kjt}^A > 0 \qquad k \notin J^P, j \in I_k^B, t$

$L_{y_{kjt}^A} = -c_{jit}^s + \theta_{jt} + \pi_{it} \leq 0, \quad = 0 \; if \; y_{jit}^p > 0 \qquad \forall j \in J^P, i, t$

$L_{y_{jt}^s} = -u_{jt} + v_{jt} + \lambda_{jt}(z_{jt} - 1) \leq 0, \quad = 0 \; if \; y_{jt}^s > 0 \qquad \forall j \notin J^P, t$

$L_{y_{jt}^s} = -u_{jt} + \theta_{jt} \leq 0, \quad = 0 \; if \; y_{jt}^s > 0 \qquad \forall j \in J^P, t$

$L_{w_{jt}} = E_{jt} - a_{sj} \leq 0; \quad = 0 \; if \quad w_{jt} > 0 \qquad \forall s = -T^0, ..., 0$

$L_{w_{jt}} = E_{jt} - s_{sj} \leq 0; \quad = 0 \; if \quad w_{jt} > 0 \qquad \forall s = 0, ..., T$

Since z_{jt} takes on integer values, the lagrangian is not differentiable everywhere with respect to z_{jt}. Therefore we use the difference in the lagrangian value over $z_{jt} = 0$ and $z_{jt} = 1$.

$L_{z_{jt}=1} - L_{z_{jt}=0} = -c_{jt}^A + \lambda_{jt} y_{jt}^s \geq 0, \quad if \; z_{jt} = 1 \qquad \forall t, j \notin J^P$

where $L_{z_{jt}=1}$ and $L_{z_{jt}=0}$ are the values of the lagrangian function if location *j* is opened and closed in period *t* respectively.

The first order conditions may be re-arranged as:

$$R'(y^{d}_{it}) = \pi_{it} \qquad (3.13)$$

$$a_{sj} \geq u_{jt} v_{sjt} + s_{jt} - c_{sjt} \qquad \forall\, j, s = -T^{0}, \ldots, 0 \textit{ and } t = 0, \ldots T \qquad (3.14)$$

$$s_{sj} \geq u_{jt} v_{sjt} + s_{jt} - c_{sjt} \qquad \forall\, j, s = 0, \ldots, T \textit{ and } t = s + z, \ldots T \qquad (3.15)$$

$$\lambda_{jt} y^{s}_{jt} \geq c^{A}_{jt} \qquad \forall t, j \notin J^{P} \qquad (3.16)$$

$$v_{kt} \geq u_{jt} - c^{s}_{kjt} \qquad k \notin J^{P}, j \in I^{B}_{k}, t \qquad (3.17)$$

$$\theta_{jt} \geq \pi_{it} - c^{s}_{jit} \qquad \forall j \in J^{P}, i, t \qquad (3.18)$$

$$u_{jt} \geq v_{jt} + \lambda_{jt}(z_{jt} - 1) \qquad \forall j \notin J^{P}, t \qquad (3.19)$$

$$u_{jt} \geq \theta_{jt} \qquad \forall j \in J^{P}, t \qquad (3.20)$$

$$a_{sj} \geq E_{jt} \qquad \forall s = -T^{0}, \ldots\ldots, 0 \qquad (3.21)$$

$$s_{sj} \geq E_{jt} \qquad \forall s = 0, \ldots\ldots, T \qquad (3.22)$$

Economic Interpretation of the First Order Conditions

The first equation, Equation (3.13), implies that marginal revenue of a wood product at the mill equals the price of the product. The right hand side of Equation 3.14 is the value of the wood at rotation minus the cost of growing plus the value of the next rotation for every rotation $t = s+z, \ldots, T$. Equation (3.14) implies that the dual variable a_{sj}, which is interpreted as the land value of type j if born in period s, is bounded from below by the expression on the right for every t from $s+z$ to T. This means the land value is at least equal to the rotation t that gives the maximum value. The interpretation of Equation (3.15) is similar to Equation (3.14). This condition is interpreted as the value of the wood at rotation minus the cost of growing plus the value of the next rotation for every rotation $t = s+m, \ldots, T$. This equation represents a generalization of the simple Faustmann (1849) rotation model. The generalization is that prices may vary over the rotation period and in subsequent rotation periods. Also this shows how the forest level model links to the simple one stand optimal rotation model. The simple recursive structure of the equations lends itself to solution by backward dynamic programming. Equation (3.21) means the bareland value of the existing stand for any analysis area is at least as great as its value if left as ending inventory. The meaning of Equation (3.22) is that the bareland value of the regenerated stand for any analysis area is at least as great as its value if left as ending inventory.

Equation (3.16) provides the criteria to determine whether to access a location or not. This equation means that if the value of wood crossing over the location j in period t is at least equal to the access cost, then the location should be opened. On the other hand if it is less, the location should remain closed.

Equations (3.17) to (3.20) set out the optimality conditions for i) determining the roadside price of wood for each stand in both permanently accessed areas and unaccessed areas, and ii) determining the optimal path from unaccessed areas onto the existing road network. Equations (3.17) and (3.19) deal specifically with unaccessed areas. To interpret 3.17, we note that v_{kt} is the net value of wood at location k and u_{kjt} is the net value of wood at an adjacent location j. This condition therefore says that the net value of wood at location k is equal to the maximum value over all shipping alternatives from k (that is the value at each adjacent j minus the shipping cost to j). This implies the wood should be moved to the location that gives the highest net value. Equation 3.18 simply says the value of wood at location j is equal to the mill price minus transportation costs, whilst Equation (3.19) means the net value of wood at j is equal to the value of wood at j minus the access cost adjustment. The access cost adjustment is the shadow price (λ_{jt}) on the access constraint, which can be interpreted as the average cost/m^3 of opening up a closed location. If a location is opened, then the access cost adjustment does not apply. But for closed locations, this average cost has to be subtracted from the value of the wood at location j to give its net value. Equations (3.17) and (3.19) form a recursive system of equations that forms the basis for the solution algorithm that determines optimal route of wood flow across unaccessed areas discussed in the next section.

The optimal rotation decision resulting from the application of Equations (3.13) and (3.14) are similar to the Faustmann rule. However, the optimal timing of harvests in this model is modified by the presence of Equation (3.16), which modifies the prices to shift the harvest cycle away from certain periods where there are few adjacent stands (i.e., stands within the same location) being harvested. What Equation (3.16) essentially does is to impose a penalty for harvesting in periods where little or no other harvesting takes place. This penalty (λ_{jt}) is derived from the fixed cost of access at a location level and from the volume of wood flowing over the location. The optimal harvest rule is modified in the sense that some stands that would have been harvested in period *t* under a zero cost model ($\lambda_{jt} = 0$) will be harvested at a different time if $\lambda_{jt} > 0$ and vice versa. For example, if a stand has few stands nearby that are close to optimal rotation or that provide large revenue then the chances of it being harvested are reduced because the entire road cost would rest on that stand. In this case, the value of the penalty (λ_{jt}) will be high. On the other hand, a stand that has more stands near the optimal rotation age will tend to have a lower penalty and so has high chances of being harvested.

A significant effect of a positive λ_{jt} is that it will tend to concentrate harvests over the landscape. This implies that trees of marginal value that are located close to high value timber are more likely to be harvested once the location is opened. It is also expected that access costs will act as an incentive for reduced frequency of harvesting in a given location. When access costs are included, the number of times in the planning horizon that the location is harvested should be reduced. Although this model does not include non-timber values, it is important to point out that access may significantly affect these values as well. Detailed analyses of these effects are deferred to the Chapter 4.

Solution Technique

We define the number of decision variables and constraints to the access problem using a planning horizon of 100 years with 10 planning periods, a minimum rotation of 40 years, three regeneration prescriptions per stand (natural regeneration, basic, and intensive silviculture), a total of 6,156 stand types, 18,883 analysis areas, and approximately 6 shipping alternatives for each stand. With these assumptions, the resulting model has approximately 2.6 million decision variables and about 96,000 constraints. Details of the calculations of the number of constraints and decision variables are given in Appendix I (Tables A1 and A2). It is important to emphasize that there are 3,850 integer decision variables and the same number of integer constraints in this formulation, which in a branch and bound algorithm would have 2^{3850} combinations of solutions although the number of combinations that actually need to be explored is somewhat less than this because of the constraints on sequencing. Even with today's fast computers this will be an extremely difficult problem to solve using the branch and bound technique. Another advantage of this formulation is that the stand level management decisions do not change from that presented in Chapter 2. This is easily verified by examining the similarity in interpretation between Equations (2.18) and (2.19) of Chapter 2 and (3.14) and (3.15) in this chapter.

The model was solved using a variant of the dual decomposition algorithm proposed by Hoganson and Rose (1984). Only the general outline is discussed here. The detailed algorithm for the solution is given in Appendix IV. First, using a geographic information systems map of the study area, we defined locations on the map that are currently accessible by major roads (primary and secondary paved roads). These are referred to as permanently opened/accessed locations (POLs) and are considered opened in each time period throughout the planning horizon. All other areas are considered closed at the beginning of each model run, and these closed areas are sorted according to how far away they are from the POLs, starting from those locations that are directly adjacent to POLs, those that are one location away, etc. There were 192 POLs ($z_{jt} = 1$), and 385 closed ($z_{jt} = 0$) locations at the beginning of a model run. The solution algorithm for this problem described below is based on the first order conditions of the lagrangian function derived above. The algorithm begins by solving each stand level problem using initial guesses at the shadow prices for each forest wide constraint for both POLs and initially closed locations.

For all POLs, we determined the price of wood in each location using Equation 3.18, which implies that the value of wood at location *j* is equal to the mill price minus transportation costs. That is, $\theta^s_{jt} = \pi_{it} - c^s_{jit}$. The maximum of these θ^s_{jt} determines which mill should receive wood from which POL.

To determine the value of wood in the unaccessed locations, we defined subdestinations on the way to the mills at locations that are permanently accessed and adjacent to areas that are not accessed. Also, each unaccessed location is a subdestination. This means that the subdestinations accumulate volumes not only from harvest within their associated supply locations but also from other locations that ship wood through those subdestinations. To determine the direction in which wood should be shipped between locations, we use the first order condition given by Equation (3.17). From this equation, $v_{kt} = u_{jt} - c^s_{kjt}$ we note that v_{kt} is the net value of wood at location k and u_{jt} is the net value of wood at location j. This condition therefore says that the net value of wood at location k is equal to the maximum value over all shipping alternatives from k (that is the value at each adjacent j minus the shipping cost to j). This implies the wood should be moved to the location that gives the highest net value.

The price of wood in each subdestination was calculated by solving iteratively the dynamic programming formulation given by Equation 3.23 (which is a combination of Equations 3.19 and 3.17)

$$u_{jt} = \max_{k \in I_j^B} \left\{ u_{kt} - c^s_{jkt} \right\} + \lambda_{jt}(z_{jt} - 1) \qquad (3.23)$$

Equation 3.23 is solved using the algorithm given in Appendix IV. Once the prices of wood in the POLs and the initially closed locations (the θ_{jt}'s and u_{jt}'s), are estimated, we use these estimates to solve the stand level management problem given by Equations 3.14 and 3.15. These stand level decisions include harvest timing for initial and subsequent harvests, mill destination for each timber type, and regeneration options. After all of the stand level problems are solved, the volume flows implied by the harvest timing and transport options are added up and compared to the demand constraint levels. If the flows deviate from the constraint levels and mill demand levels then the shadow prices are adjusted using simple intuitive shadow price adjustment procedures described by Hoganson and Rose (1984) and modified by Hauer (1993).

At an optimal solution λ_{jt} from Equation 3.16 must satisfy $\lambda_{jt} \geq c^A_{jt} / y^s_{jt}$. The right hand side of this equation represents a lower bound on λ_{jt} if an area is opened. If λ_{jt} is less than c^A_{jt} / y^s_{jt} , then an area cannot be opened. The interpretation of the dual variable λ_{jt} , given Equation 3.16, represents the net value of wood per cubic meter (net of transport and harvest costs). If the net value per cubic meter is greater than the access cost per cubic meter then it makes sense to ship wood over the location.

The shadow prices on the access constraints are adjusted based on Equation 3.5, which is given as: $y^s_{jt} \leq y^s_{jt} z_{jt}$. This equation basically states that wood cannot be harvested from or transported through an unaccessed location. Therefore, after solving the stand level problems, the algorithm checks all locations and calculates a deviation for the constraint as $dev_{jt} = y^s_{jt} - y^s_{jt} z_{jt}$. The deviation is either positive or zero. A positive deviation means that wood is harvested from or transported through a location that is not accessed ($z_{jt} = 0$). In this case, the shadow price on the constraint is adjusted upward for that location in the next iteration. That is, for $dev_{jt} = y^s_{jt} - y^s_{jt} z_{jt} > 0$, $\lambda^1_{jt} = \lambda^0_{jt}, + f^g(dev_{jt})$, where λ_{jt} is the shadow price on the access constraint. The function f^g is piecewise linear (Figure 3.1) which gives price adjustments as a function of the deviation (dev_{jt}). The function gives small price changes for large deviations and large price changes for small deviations. This is because the areas that have lots of wood flowing over them need smaller λ_{jt} to justify transport of wood over them.

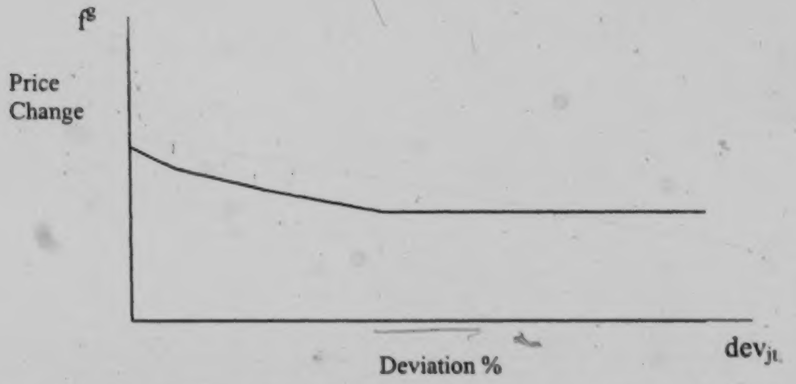

Figure 3.1. The relationship between the deviations and price adjustments for the access constraint for deviations greater than zero.

If the $dev_{jt} = y^s_{jt} - y^s_{jt} z_{jt} = 0,\ y^s_{jt} > 0,\ \ z_{jt} = 1$, then the price changes are based on $\lambda^1_{jt} = \lambda^0_{jt} + f^0\left(\frac{c^A_{jt}}{y^s_{jt}} - \lambda^0_{jt}\right)$. The deviation is always negative since for the place to be opened its λ_{jt} must be greater than c^A_{jt} / y^s_{jt} . The shape of f^0 is shown in Figure 3.2.

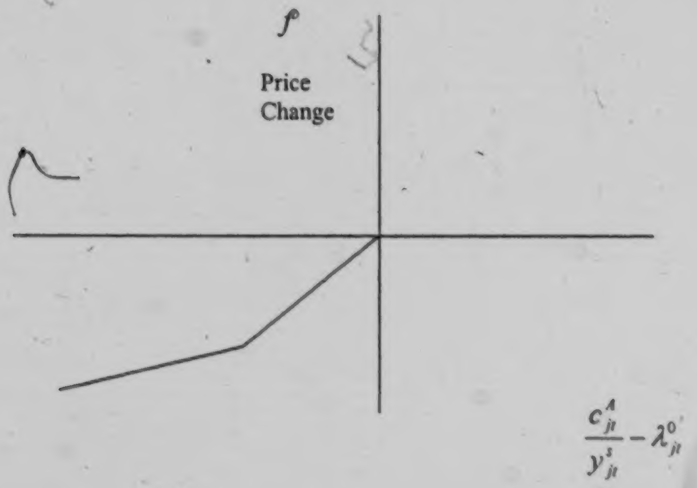

Figure 3.2. Shape of the price adjustment function for opened locations and zero deviations.

The price changes are large if the difference between the λ_{jt} in a particular iteration and the previous iteration, $\frac{c^A_{jt}}{y^s_{jt}} - \lambda^0_{jt}$, is large, and vice versa.

After the price changes, we update the database of opened and closed locations. The decision as to whether to open a location or not is based on Equation 3.16 if $\lambda_{jt} y^s_{jt} \geq c^A_{jt}$ then $z_{jt} = 1$. On the other hand, if $\lambda_{jt} y^s_{jt} < c^A_{jt}$ then $z_{jt} = 0$. This means that if the value of wood crossing over the location *j* in period *t* is at least equal to the access cost, then the location should be opened. Otherwise, the location should remain closed. This process continues until the demand and access constraints are satisfied with a reasonable tolerance. The result of this process defines continuous roads for transporting wood from each location to the mill that maximizes the net present value. This is consistent with a minimum cost path for constructing and transporting the wood from each location to the mills at Drayton Valley.

RESULTS

Model Performance

The program for the two model runs described in this paper was coded in C and implemented on a personal computer with a Pentium III 500 Mhz microprocessor. The first model is called the Baserun, in which the access costs are computed as an average and then added to the marginal harvesting costs at the stand level. What this means is that one can access a stand anywhere without explicitly building an access route. The second run is called the Access Model, and includes a fixed cost of constructing a road from one location to the next adjacent location of $20,000. This amount was determined based on estimates of the average road construction cost per cubic meter of wood harvested in the Weyerhaeuser FMA. Therefore the Access Model means that no location within the unaccessed portion of the forest can be assessed without first building an access route. The cost of accessing stands within locations was added as an additional harvest cost. In both models, there are two demand centers (a sawmill and an oriented strand board (OSB) mill) both located at Drayton Valley. The maximum demand of final product at the sawmill and OSB mill are 90,000m^3/year and 200,000m^3/year respectively, with maximum price/m^3 of final product set at $300 for lumber and $100 for OSB. These maximum price levels were set based on current estimates of the prices of lumber and OSB. Production costs include harvesting, transportation, as well as milling costs. The criteria for determining when to stop a run was based on the average absolute percentage deviation of the end product from the target demand for each mill, their distribution around the demand, and the number of locations violating the access constraints. The model takes 6 minutes (about 120 iterations) to arrive at a solution if road access costs are zero, and about 20 minutes (about 400 iterations) if access costs are $20,000 per location. The maximum deviation of each end product from target demands for any period for the model to converge was set at 3%. Table 3.1 shows that all the model runs produced satisfactory results, with all average mill deviations less than 2%. The model also performed very well in terms of the integer constraints, as there were no violations of this constraint when the model converged. Furthermore, the differences between objective and lagrangian function values, which measure model convergence, were 0.0005% and 0.0003% for the Baserun and Access Model respectively.

Effect of Access Costs on Marginal Costs

The results show a systematic agreement with our theoretical expectations regarding the impacts of access on the harvest schedule. The results in Table 3.1 reveal that in the Baserun, wood was harvested from all locations except 49 in at least one period throughout the planning horizon. On the other hand, when access costs are $20,000/location, 76 locations are unaccessed. Positive road building costs therefore tend to reduce the number of locations accessed and concentrate harvesting to only locations where it is economically profitable to do so. This is further shown in Table 3.1 by the fact that a smaller area was harvested in the Access Model, than in the Baserun. Economic intuition suggests that the access costs in the Access Model might be considered a fixed cost over all the stands within a location. Hence, there will be incentives to harvest more per ha than when the access cost is treated as a marginal harvesting cost in terms of $/m^3 as in the Baserun. Therefore more stands per location were harvested and fewer areas were accessed in a particular period in the Access Model than the Baserun. Table 3.1 also shows that the NPV of the Access Model is about $3.97 million less than the NPV of the Baserun. The small difference in the NPV may be because road access costs are incorporated as a smooth marginal cost in the harvesting costs for the Baserun.

Table 3.1. Comparisons of the two models in terms of the deviations from mill demands, areas harvested, and the net present values, for the planning horizon.

Model run	Average absolute deviations (%) from Demand Constraint		Number of locations not accessed	Total Area harvested (ha)	Net present value (10^6 $)
	Sawmill	OSB mill			
Baserun	1.395	1.323	49	310,909	596.70
Access Cost	0.590	0.864	76	293,156	592.73
Difference			27	17,753	3.97

The shadow prices on the mill demand constraints for lumber and OSB mills for both models are shown in Figure 3.3. These shadow prices are the marginal costs of regenerating, harvesting, accessing, transporting the wood to the millgate, and milling for each mill. Shadow prices (marginal costs) for the sawmill are significantly higher in the Access Model than in the Baserun and both are increasing over the planning horizon. This is consistent with the lower NPV for the Access Model than the Baserun shown in Table 3.1. We also observe that the difference between the marginal costs of the two models becomes greater in the later planning periods. The increasing marginal costs signal the scarcity of conifer wood in the permanently accessed locations, and suggest that more locations have to be accessed in order to meet the mill demands. The data presented in the first row of Table 3.2 show that more areas are accessed later in the planning horizon. However, for the OSB mill, the marginal costs for the Access Model are only slightly higher than the Baserun and both decrease slightly over time (Figure 3.3).

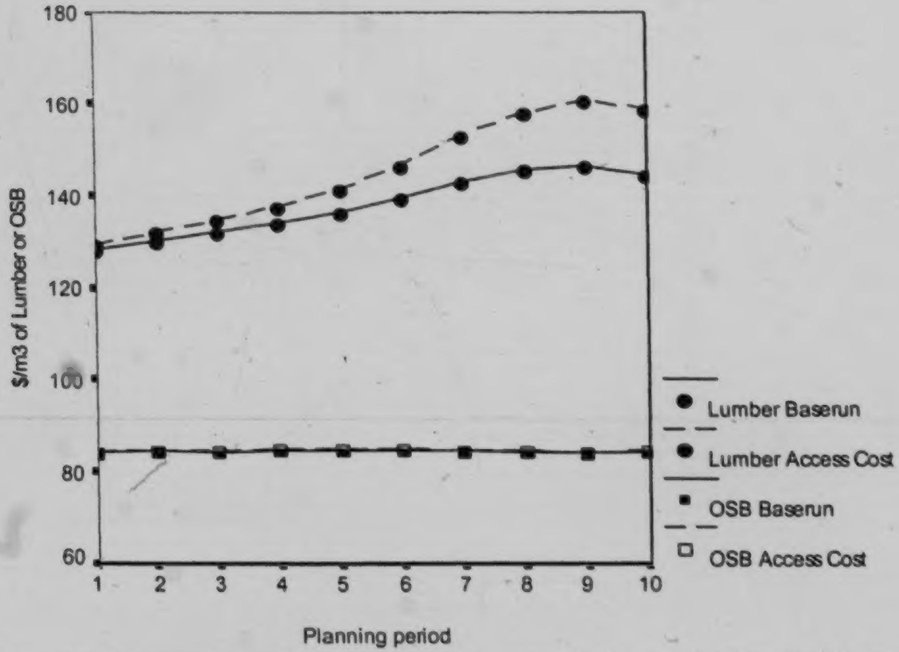

Figure 3.3. Comparison of the shadow prices for lumber and OSB for the Baserun and Access Model.

The reason for the small difference in the marginal costs of the OSB mill between the two models may be that the demand specified for aspen is low, and so there is enough aspen on the land base to satisfy this demand at low marginal costs. The decreasing costs due to the abundance of aspen off-set the increasing marginal cost effect of including the access cost. It may also be the case that since both species types are harvested once a location is opened, the high value of conifer is subsidizing the road construction costs for aspen.

Impacts of Access Costs on Road Development and Harvest Schedule

Analyses of the harvest schedule for each mill revealed how the transport destinations for wood in each location change under the two models in the first and second planning periods. Figures 3.4 and 3.5 show the distribution of the 192 permanently accessed locations, and wood flow from non-permanently accessed locations for the first and second periods in the planning horizon. The woodflow was in line with our expectations that wood from non-POLs will move to the nearest POLs in order to minimize the costs of road access.

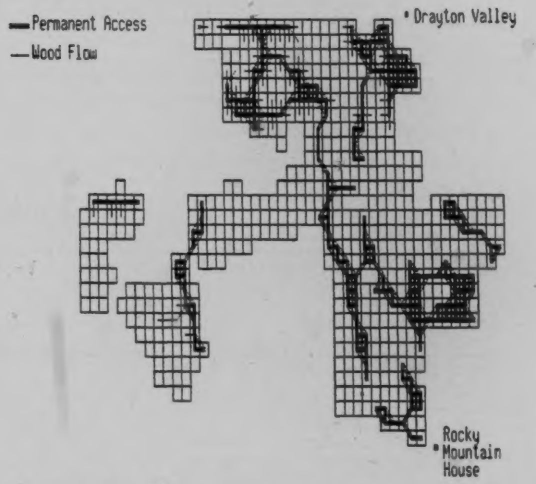
Permanent Access
Wood Flow
Drayton Valley
Rocky Mountain House

Figure 3.4a. Permanent access and directions of woodflow for the Baserun in period 1.

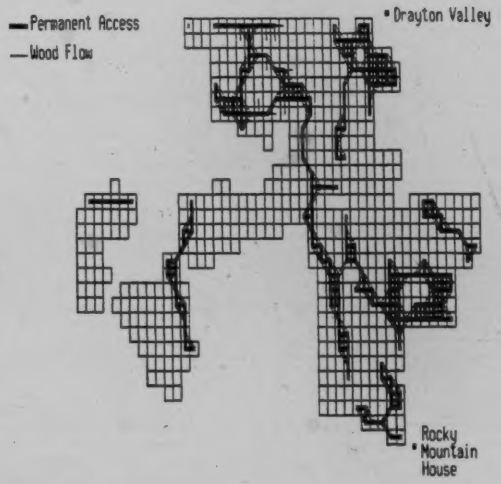
Permanent Access
Wood Flow
Drayton Valley
Rocky Mountain House

Figure 3.4b. Permanent access and directions of woodflow for the Access Model in period 1.

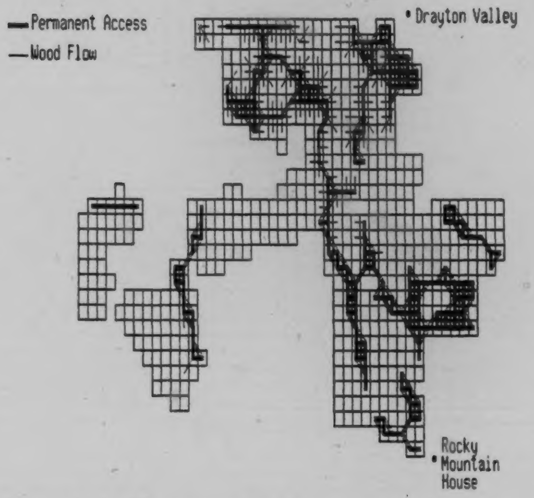
Permanent Access
Wood Flow
Drayton Valley
Rocky Mountain House

Figure 3.5a. Permanent access and directions of woodflow for the Baserun in period 2.

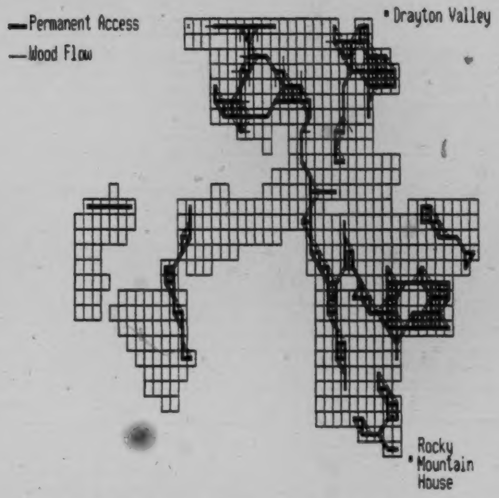
Permanent Access
Wood Flow
Drayton Valley
Rocky Mountain House

Figure 3.5b. Permanent access and directions of woodflow for the Access Model in period 2.

It is also obvious that almost all the non-POLs accessed were concentrated around permanent access. The lack of explicit access cost in the Baserun resulted in more non-

POLs accessed than in the Access Model, whilst the inclusion of access costs shifts the harvesting onto the permanently accessed land base and spreads harvesting to the south as well.

The harvest schedules, shown in Figures 3.6 – 3.9, reveal the differences in the supply locations that are harvested and how much is harvested in each supply location during the first two periods in the planning horizon. The schedules for both periods and models show that more locations with permanent access were harvested than those that had no permanent access. The Baserun consistently harvested more non-POLs than the Access Model, which implies that the cost of access provision was a disincentive to harvest wood from locations that are not permanently opened. Furthermore, the fact that access costs are fixed over multiple stands means there is an incentive to concentrate the cut rather than spread it out. It was also observed that all non-POLs opened under the Access Cost model in the first two periods were locations close to Drayton Valley, whilst the non-POLs opened in the Baserun were more spread out and farther away from Drayton Valley. Again, this is an expected result, as the road building costs will make stands far away from the demand center uneconomic to open. A comparison of the number of locations harvested in period 1 and period 2 shows that for both models and location types, the number of harvested locations were higher in period 2 than period 1, and many more locations farther away from the demand center were harvested. Finally, we can conclude from a comparison of the pattern of harvested locations between POLs and non-POLs in the Access Cost Model that it was cheaper to harvest and transport wood from POLs that were far away from Drayton Valley than to open up locations close to the demand center. These results therefore indicate that the nature of the distribution of harvests on the whole landscape is highly dependent on the initial layout of permanent access on the landscape.

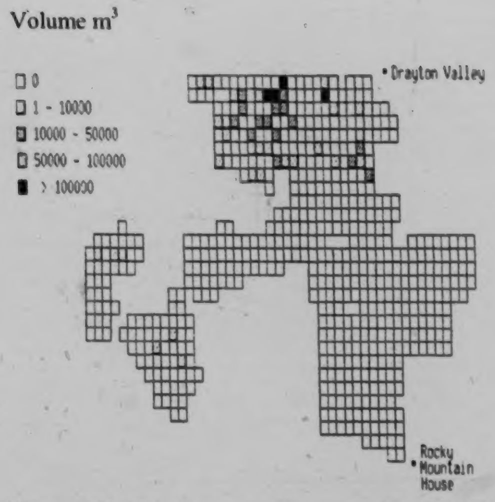
Volume m^3
0
1 - 10000
10000 - 50000
50000 - 100000
> 100000
Drayton Valley
Rocky Mountain House

Figure 3.6a. Wood procurement zone from Non-Permanently Opened Locations for the Baserun in Period 1.(Total number of locations harvested = 47).

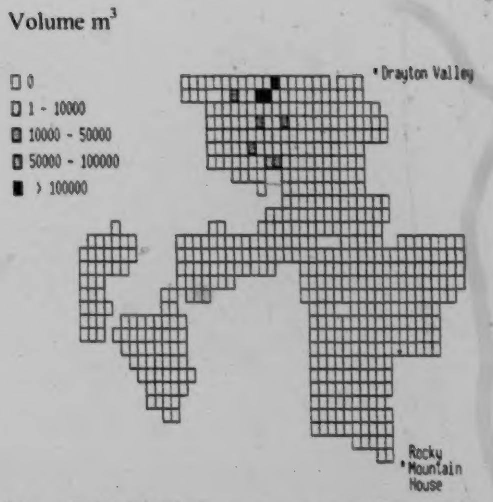
Volume m^3
0
1 - 10000
10000 - 50000
50000 - 100000
> 100000
Drayton Valley
Rocky Mountain House

Figure 3.6b. Wood procurement zone from Non-Permanently Opened Locations for the Access Cost Model in Period 1. (Total number of locations harvested = 9).

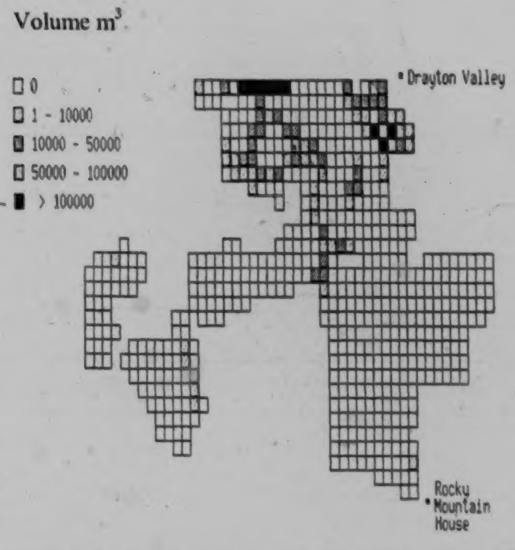
Volume m^3
0
1 - 10000
10000 - 50000
50000 - 100000
> 100000
Drayton Valley
Rocky Mountain House

Figure 3.7a. Wood procurement zone from Permanently Opened Locations for the Baserun in Period 1.
(Total number of locations harvested = 88).

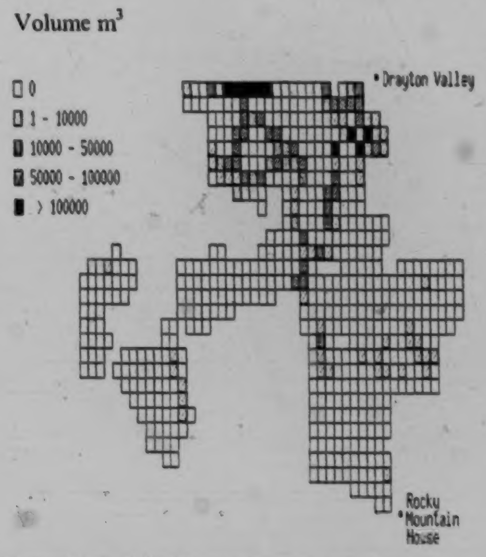
Volume m^3
0
1 - 10000
10000 - 50000
50000 - 100000
> 100000
Drayton Valley
Rocky Mountain House

Figure 3.7b. Wood procurement zone from Permanently Opened Locations for the Access Cost Model in Period 1. (Total number of locations harvested = 109).

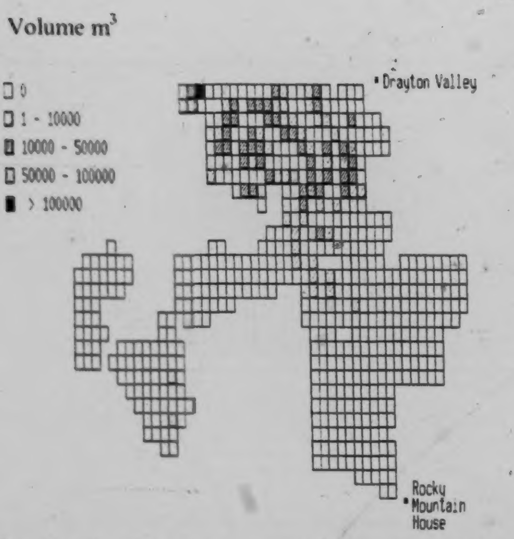
Volume m³
☐ 0
☐ 1 - 10000
▨ 10000 - 50000
▨ 50000 - 100000
■ > 100000
• Drayton Valley
Rocky
• Mountain
House

Figure 3.8a. Wood procurement zone from Non-Permanently Opened Locations for the Baserun in Period 2. (Total number of locations harvested = 89).

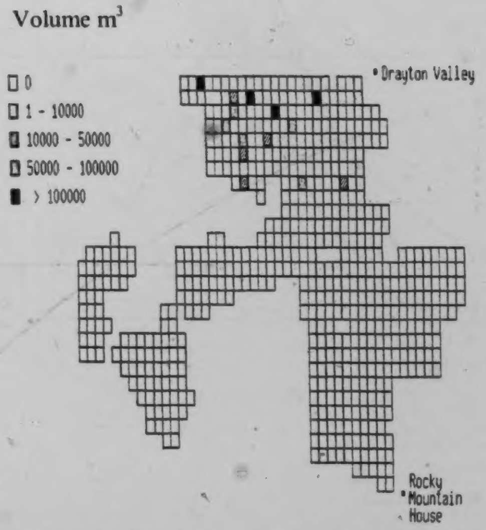
Volume m³
☐ 0
☐ 1 - 10000
▨ 10000 - 50000
▨ 50000 - 100000
■ > 100000
• Drayton Valley
Rocky
• Mountain
House

Figure 3.8b. Wood procurement zone from Non-Permanently Opened Locations for the Access Cost Model in Period 2. (Total number of locations harvested = 14).

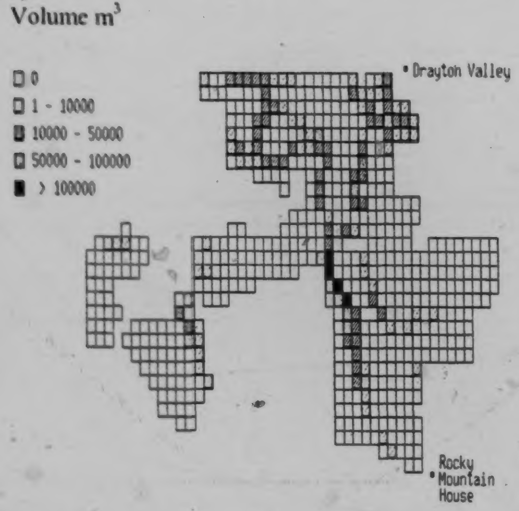
Volume m³
☐ 0
☐ 1 - 10000
▨ 10000 - 50000
▨ 50000 - 100000
■ > 100000
• Drayton Valley
Rocky
• Mountain
House

Figure 3.9a. Wood procurement zone from Permanently Opened Locations for the Baserun in Period 2.
(Total number of locations harvested = 126).

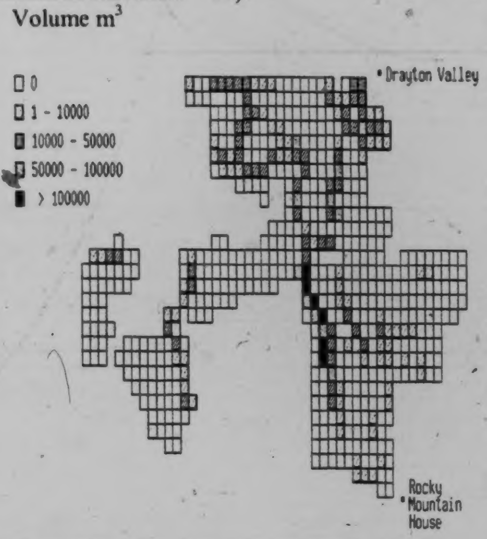
Volume m³
☐ 0
☐ 1 - 10000
▨ 10000 - 50000
▨ 50000 - 100000
■ > 100000
• Drayton Valley
Rocky
• Mountain
House

Figure 3.9b. Wood procurement zone from Permanently Opened Locations for the Access Cost
Model in Period 2. (Total number of locations harvested = 153).

Table 3.2 shows the number of locations opened in each period and changes in the average area harvested. The number of locations opened for the Access Model increased from 9 in the first period to a high of 74 in the ninth period. In the first few periods, there is enough wood in the permanently accessed areas to meet mill demands, and so fewer unaccessed locations are opened. But as these are harvested, and given the minimum 4 periods between harvests, more locations need to be accessed. This is the main reason for the divergence in the marginal cost at the mills over time. The average area harvested per location is higher for the Access Model in all periods than the Baserun. This is to be expected since large volumes have to be harvested to justify opening up the locations in the first place. This shows intensive harvesting in each location when access costs are positive as a result of the extra cost of accessing more stands. In fact, on average, approximately 52 ha more area are harvested per location in each period in the Access Model than the Baserun.

Table 3.2. Comparison of number of locations opened, areas harvested, and the shadow prices on the access constraint by period for the Baserun and Access Model.

	Planning period									
	1	2	3	4	5	6	7	8	9	10
No. of closed locations opened	9 (49)*	14 (90)	33 (128)	43 (147)	61 (222)	52 (238)	68 (242)	68 (262)	74 (295)	69 (222)
Area harvested ('000 ha)	29 (28)	23 (23)	23 (23)	23 (24)	25 (24)	24 (26)	24 (25)	35 (36)	49 (57)	47 (59)
Average area harvested/ location (ha)	244 (208)	140 (108)	120 (83)	131 (81)	136 (67)	140 (72)	143 (71)	149 (91)	190 (124)	203 (148)
Cost/m^3 of opened locations (Average λ_1)	0.50	0.38	0.41	0.59	0.69	0.57	0.52	0.96	1.02	1.28
Cost/m^3 of closed locations (Average λ_1)	4.94	4.87	5.16	5.75	6.58	8.49	9.64	11.81	11.62	12.13

*Note: Numbers in parenthesis are the Baserun Model equivalents.

Shadow Prices on the Access Constraint

The shadow prices associated with the access constraint [Equation 3.5] for each period are given in the last two rows of Table 3.2. The shadow price, λ_{jt}, can be interpreted as either costs or benefits of opening up a location. It represents the value of being able to relax constraint (3.5) by one unit – in other words, be able to harvest one more m^3 of wood without actually having to access the location. In this sense λ_{jt} represents a net value of wood (net of transport and harvest costs) per cubic meter coming from a particular location. This is the interpretation implied by Equation 3.16, where λ_{jt} represents the amount that wood would have to be worth to justify opening an area. The other way of interpreting it is that the λ_{jt} represents the marginal cost of having to access the area in order to harvest another m^3 of wood. This interpretation of λ_{jt} is consistent with Equation 3.19, where λ_{jt} is the marginal cost of accessing a location for purposes of harvesting a m^3 of wood. From Equation 3.19, it implies that the presence of λ_{jt} makes the marginal cost of harvesting a small amount of wood in a closed location, which a timber harvest-scheduling model without the fixed access cost would schedule for harvest, very high. This marginal cost is subtracted from the marginal value of wood, which eventually squeezes the volume flow from the locations with small wood flows to zero.

The shadow prices on the access constraint (λ_{jt}) increase over time (Table 3.2). This result implies that the marginal value of the wood is increasing over time, which would be consistent with lower wood flows across each location in the future. Lower wood flows require higher marginal wood values in order to justify the higher cost/m^3 road building cost that occurs. The shadow prices of the opened locations are significantly lower than those of closed locations because the marginal access cost of increasing the wood flow in opened locations is in fact zero. The large differences in the shadow prices between the opened and closed areas show that most of the closed areas have small volumes of harvestable wood and so they are not profitable to open if access costs are included. Therefore, given the assumptions of the model, the prices of wood products in the closed locations have to increase by the average amounts given in the last row of Table 3.2 to make these locations economic to open.

The distribution of locations according to the number of periods they were opened throughout the planning horizon is given in Table 3.3. The table provides information on the number of non-POLs that were opened 0, 1, …,10 times during the planning horizon. It shows that for the Access Model, no location was opened more than 5 times, whilst for the Baserun, the maximum number of times locations were opened was 10.

Table 3.3. The distribution of locations according to the number of times opened during the planning horizon.

	Number of times opened										
Model Type	0	1	2	3	4	5	6	7	8	9	10
Baserun	49	14	26	28	30	55	51	55	44	24	9
Access Model	76	163	117	24	3	2	0	0	0	0	0

*Note: These frequencies do not include the 192 permanently opened locations.

The inclusion of access costs considerably reduces the frequency with which a given location is opened. This result could be useful if provision of access is important for non-timber values in the forest, especially if recent evidence of harvesting activities in the FMA negatively impact on recreation or other non-timber benefits.

DISCUSSION

The approach presented in this paper holds great promise for use in practice to examine the impacts of road construction costs on long-term timber supply. The case study shows that the model we developed is capable of addressing the problem of jointly solving the forest management and access development problem. Solving large mixed integer linear programming problems that include silvicultural and access considerations is very difficult for a large number of integer variables, and has been investigated for a long time. The method used here provides a least cost strategic access plan for constructing roads with large temporal and spatial detail. Furthermore, with integer variables in the access cost model, the model took only about 20 minutes to converge to a solution.

The results of this paper are consistent with economic theory and intuition regarding the effects of incorporating access costs in timber management scheduling. One of the effects that access costs has on scheduling is on the timber values. Timber values with and without access costs were estimated using the NPV of the Access Model and Baserun respectively. A comparison of the NPVs between the Baserun and Access Model show that the difference was small. The reasons for the small difference could be because road access costs are incorporated as a marginal cost in the harvesting costs for the Baserun. The economic implication of the closeness of the NPVs however, is that given the assumptions in our model, future access development in the FMA will not reduce timber revenues significantly.

Another important application of this approach lies in its ability to estimate the shadow prices of all constraints in the model. The shadow prices on the demand constraints indicate the marginal costs of producing the final products. Marginal costs for the sawmill were higher than the OSB mill and increased over time when access costs were $20,000/location, compared to the scenario with zero access costs. Therefore when access costs are high, marginal costs increase accordingly, and so fewer locations are harvested. For the OSB mill, marginal costs under the Baserun and Access Model remained relatively constant and decreased slightly over the planning period. This may suggest that the maximum demand level set for the OSB mill is low compared to what the forest can sustain. Therefore, the harvest of aspen may need to increase in future. Secondly, the shadow prices on the access constraint give us the average costs of road construction per cubic meter of wood harvested, if the location is open. This information is a useful indicator of how expensive it is to invest in road development, and whether it is profitable to do so. The shadow price for closed locations indicates the minimum dollar value that should be paid for cubic meter of wood to make it profitable to open the location. This information is important in determining whether road construction to unaccessed locations make economic sense. Hence, the model provides a way of relating the wood production to marginal costs and values of timber, which can be compared to expectations of future timber prices. This provides valuable information for supply planning and current planning in silvicultural and road investment expenditures. For example, by conducting sensitivity analyses, it is possible to determine how much wood can be profitably harvested given a fixed road budget, or given a fixed demand, how much it will cost to build roads to satisfy that demand.

The schedules for the two models for the first two periods in the planning horizon also revealed very important implications of considering access costs in long-term timber supply analyses. The harvesting pattern was contrary to a common sense expectation that locations that have permanent access will necessarily be harvested before opening up any unaccessed areas. In the Baserun, locations without permanent access and close to the demand center (Drayton Valley) were accessed and harvested before POLs that were far away from the demand center. In contrast, POLs that were far away from the demand center were harvested before opening up locations that were close to Drayton Valley in the Access Model. As the marginal values of final timber products rise in later periods of the planning horizon, the value of wood in the closed locations increase accordingly, and so makes it profitable to open up adjacent closed areas. An important conclusion is that the inclusion of access costs determines both the initially harvestable stands and subsequent road development. Access development during the planning horizon is also dependent on the layout of the permanent access within the forest, as road construction spreads from these POLs to adjacent closed locations.

The inclusion of positive access costs has the tendency to concentrate forest management activities to fewer locations, and increase the area harvested per accessed location. This is easily verified from a comparison of the areas harvested/location, as well as the number of locations accessed under either model given in Table 3.2. If forest management involves non-timber benefits (e.g., recreation or hunting), then the number of locations accessed becomes an important consideration. For example, access to areas has most often shown to negatively affect hunter utility of hunters (e.g., McLeod *et al.*, 1993). In this case, fewer access areas due to positive access costs will tend to increase the non-timber benefits. Of course, there may be situations under which provision of access increases non-timber values. It is therefore important that long-term analysis of timber supply incorporate access costs as this impact on not only the timber values, but the non-timber values as well.

Although this model investigated a specific example of a forest access problem, it is possible to evaluate other management problems related to access and to make the solution method more efficient. For example, the present set up of the model allows a location to be opened for one period (10 years), after which it is either closed or opened again in the next period. However, if a two- or three-pass harvest system is used, it may require that areas that are opened in one period remain opened for the next two or three periods. Our model further assumes that the POLs are opened and maintained at no cost. It is highly probable that the construction and maintenance costs associated with permanently opened all-weather roads will make most stands unprofitable to harvest. Also, we have not explicitly dealt with the decommissioning (closing) of roads at the end of each planning period. The cost of closing roads is currently lumped into the road construction costs. Multiple pass harvesting will require that these costs be separated, as they will occur in different time periods. Another issue that can be investigated is possible inefficiency in the cost minimizing routes that result from the model set-up and solution technique employed. The model is currently set up such that all wood from unaccessed locations has to pass through the permanently accessed locations. In a case of multiple mill destinations, it is possible for that route to be longer (and more expensive) than constructing a road directly from an unaccessed location to a mill. Finally, although the model described in this paper dealt only with timber supply, the inclusion of access provides the necessary framework to extend the model to consider non-timber values, especially recreational hunting. This is the focus of Chapter 4.

CONCLUSIONS

This study applied an optimization approach to demonstrate how an extension of the dual decomposition technique of Hoganson and Rose (1984) could be used to integrate forest scheduling and access activities. The application of this model in this paper shows the method is effective for generating optimal near feasible solutions within a short computer runtime of about 20 min. The model was applied to a large temporal and spatial forest management-scheduling problem on a Weyerhaeuser FMA near Drayton Valley, Alberta. The results reveal that inclusion of access costs concentrates forest management activities to fewer locations over the planning period compared to when it costs nothing to open up the areas. Also, positive access costs reduce the frequency with which locations are accessed during the planning horizon. The model provides important shadow price information that is useful for determining how access cost affects each demand location in the model. Using this framework, the model can be extended to deal with multiple-pass harvesting, decommissioning of roads, and non-timber benefits. The algorithm could also be extended to include variable access costs depending on factors such as terrain, availability of road materials, etc.

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CHAPTER 4

INTEGRATING A RANDOM UTILITY MODEL FOR NON-TIMBER FOREST USERS IN STRATEGIC FOREST PLANNING MODELS

INTRODUCTION

Increasing public demand for non-timber goods and services from forests has provided the motivation for forest managers to incorporate these benefits into strategic forest management analyses. Incorporating non-timber values into traditional timber supply models has the potential to change optimal rotation ages and consequently affect the timber harvest schedule. Furthermore, non-timber values affect the normal issues of choosing stands for harvest to achieve concentration in space as well as profitability based only on timber values. Concentration is desired over space because of economies associated with concentrating harvests over the landscape. Forest operations such as harvesting and road construction also affect wildlife and other non-timber values within the forest. When multiple benefits are considered, forest management activities become interdependent, and any one activity ultimately affects other values directly or indirectly.

The challenge for forest managers is not only how to incorporate these multiple values into existing forest planning models, but also to understand the nature and degree of tradeoffs that may be associated with the provision of multiple benefits. To understand these tradeoffs, we need to know the nature of the relationships between the various forest uses. Determining the relationships between the various forest uses is a task of considerable complexity, although these relationships are known to fall into three main categories: independent, complementary, or competitive (Teeguarden, 1982). When the relationships between forest uses are independent or complementary, there are no tradeoffs. However, most forest uses are more likely competitive, especially at higher levels of use, making tradeoffs inevitable. Consequently, forest managers are interested in answers to questions such as: how would timber harvest schedules change when non-timber values are included and what are the effects of incorporating non-timber benefits on timber values? In addition, whilst timber output levels are often easy to set, it is usually difficult to determine how many non-timber goods and services to produce and where, and the effect of timber management activities on non-timber users of the forests. These and other multiple-use questions are fundamental to long-term planning that ensures the sustainability of forests and allows society to derive maximum benefits from forest resources.

This chapter provides an effort to incorporate a specific type of non-timber value into a strategic forest management model in order to examine the issues raised above. We develop a utility theoretic spatial choice model and apply it to the Drayton Valley FMA of Weyerhaeuser Company. We use mixed-integer non-linear programming (MINLP) to incorporate a spatially explicitly utility function for hunter recreation values into a forest level harvest scheduling and access road development model. The resulting behavioural model was used to examine: i) how timber harvests schedules change when non-timber values are included, ii) how the welfare of hunters change with changes in timber harvest, iii) how timber values change in the presence of non-timber benefits, and iv) the effect of timber harvests and access development on hunter behaviour. Because of the importance of hunting to many communities in Alberta, this activity is used as an example to investigate the above objectives, which we believe are crucial to analyses of tradeoffs arising from conflicts in providing timber and non-timber benefits. The objective function of our model contains timber and non-timber values, subject to several constraints. Timber values are derived from the sale of timber products from the sawmill and oriented strand board (OSB) mill whilst the value of non-timber benefits are estimated using an indirect utility function for elk hunters developed for the Foothills Model Forest. This chapter contributes significantly to our understanding of the link between landscape characteristics and changes and behavioural responses by hunters and recreationists. This knowledge is important if the prediction of how hunting patterns will change in response to different management scenarios is required. It is important to emphasize that although our specification of non-timber values is not exhaustive, the methods shown here demonstrate how other behavioural models for other forms of hunting, recreation and fishing for example, could be incorporated.

Of course, this is not the first attempt to incorporate non-timber values of forests into mathematical programming models. The literature abounds with examples of different approaches to model multiple objectives of forest management using mathematical programming. The earliest and most widely used technique is goal programming, which was pioneered by Field (1973). Following the work of Field (1973), many other researchers have applied goal programming to different forest management problems, for example Kao and Brodie (1979), Field *et al.* (1980), and Diaz-Balteiro and Romero (1997). Since then, there has been an extensive body of literature addressing several forest management planning problems involving multiple objectives. Although goal programming may be adequate for examining trade-offs in forest management with multiple criteria, it suffers from the possibility of dominated solutions, and problems with goal and utility functions specification (Bouzaher and Mendoza, 1987).

Despite the popularity of goal programming, other approaches that address multiple objectives of forest management have been applied to forest problems. One such approach is the optimization of timber values with spatial and/or temporal (e.g., green up conditions) constraints to protect non-timber resources such as wildlife, water quality, aesthetics, and increased recreational opportunities (e. g., Carter *et al.*, 1997; Nelson *et al.*, 1993). Nelson *et al.*, (1993), for example, demonstrated how unit adjacency constraints could be translated into age-class profile constraints that are suitable for regulating the rate of harvest in strategic forest planning models. A potential problem associated with adjacency constraints is that the incorporation of these constraints alone does not explicitly estimate the magnitude of the value of the non-timber benefits associated with each treatment schedule.

A third approach is the use of utility functions to compare the utility from multiple objective functions, which represent the utility obtained from various forest products. The various objective functions are optimized using alternative management schedules, and the management schedule that results in the highest utility is chosen (e.g., Harrison and Rosenthal, 1986; Kilkki *et al.*, 1986; and Kangas and Pukkala, 1996)). A typical example of the utility theory approach is that presented by Harrison and Rosenthal (1986), who describe a multi-objective optimization approach for scheduling timber harvests on non-industrial private forestlands in the U. S. A. An approach to include biodiversity in calculation of multiobjective forest planning is presented by Kangas and Pukkala (1996). Their study decomposed biodiversity into measurable environmental components and included among other objectives (such as timber and amenity) in a tactical forest-planning model.

Although the above references do not attempt to be exhaustive, they clearly show the level of interest by researchers, and the range of techniques that have been employed, to incorporate non-timber values into mathematical programming timber supply models. The research presented here is conceptually similar to previous studies using utility functions. However, the method presented here differs from previous studies in a number of ways. First, most of the previous applications reported were small forest problems with limited spatial resolution. Given that large forest management problems are the rule rather than the exception in Canada, developing modeling techniques that solve large, spatially and temporally detailed models in relatively short computer time is both attractive and necessary. Secondly, we use a site choice model (random utility model), which is widely used in the literature to model consumer choice from among a discrete set of alternatives such as in recreation demand (Adamowicz *et al.*, 1997). The approach results in a behavioural model, in that as the characteristics of the forest change, the expected behaviour of hunters change and so do the expected benefits. This therefore allows for the possibility of substitution among hunting sites over time, which is endogenously driven by changes in the characteristics of the forest. Furthermore, our model explicitly considers access development, which allows us to examine the long-term impacts of access road development on non-timber benefits.

The next section of this paper gives a brief review of random utility theory, followed by a description of the utility function used in the objective function, the proposed mixed integer non-linear programming (MINLP) formulation, and the solution method. The results from an empirical application of the model to an operationally sized forest management problem is then presented, followed by a discussion of the management applications of the results, and possible extensions of the present model. The conclusions from the results are presented in the last section.

BRIEF REVIEW OF RANDOM UTILITY THEORY

This section provides a discussion of the theoretical framework for estimating the recreational benefits associated with hunting in the study area. An indirect utility function is first used to derive the maximum utility that can be obtained, the utility is then converted into a compensating variation welfare measure. Indirect utility functions characterize the maximum utility that can be achieved given prices and income. Discrete choice theory follows the same reasoning except consumption can only be in specific quantities and so allows for choices of zero or "corner solutions" in consumption. When modeling recreation demand, discrete, rather than continuous, choices are more realistic since recreationists make decisions based on going to a site or not and also because sites cannot be sub-divided infinitely or continuously. Recreational demand models typically have a finite set of alternative sites or are discrete. The choice of alternative sites is dependent on the utility, U, hunters derive from various attributes, Q, of the site:

$$U_{in} = f(M, Q_{in}, S) \tag{4.1}$$

where the utility is a function of income (M), the quality of the site or attributes describing site i as perceived by recreationist n (Q_{in}) and other socio-economic variables (S). The choice set is defined as C_n, and n is the number of alternative sites (or a subset of sites). If site i is chosen, we assume that the utility associated with visiting site i is higher than for any other site j, i. e.,

$$U_{in} > U(Q_{jn}) \qquad \forall i \neq j; \; i, j \in C_n \tag{4.2}$$

Utility in this framework is treated as a random variable since researchers do not have perfect behavioral information (McFadden 1981, Smith 1989). More formally, utility is modeled to include a systematic/observable component and a random or unobservable component:

$$U_{in} = V_{in} + e_{in} \tag{4.3}$$

where V_{in} is the systematic component of utility and e_{in} is a random element. Thus, the model is also known as a random utility model (RUM). The random element captures any unexplained factors that are not directly modeled or observed by the researcher. Since utility is formulated as a random variable, RUMs imply a probabilistic rather than deterministic outcome in choices. Thus the probability of individual n choosing site i is:

$$P_n(i) = P_r\{V_{in} + e_{in} \geq V_{jn} + e_{jn}; \quad \forall j \in C_n\} \tag{4.4}$$

where V_{in} is a conditional indirect utility function of the linear form:

$$V_{in} = \beta_1 + \beta_2 x_{in2} + \beta_3 x_{in3} +, \ldots\ldots, + \beta_k x_{ink} \tag{4.5}$$

where x_{ink} includes the attributes of the alternative sites and the social characteristics of the individual and the β's are the parameters to be estimated. Each site will have an associated conditional indirect utility function, V. The x's or attributes for equation (4.5) are distance to hunting sites, vegetation of sites, and access to hunting sites, etc. Assuming that the individual's utility function has additive error terms, e_{in}, that are independently drawn from an extreme value distribution (Gumbel), the probability condition of choosing site i is:

$$P_n(i) = \frac{e^{V_{in}}}{\sum\limits_{i \in C_n} e^{V_{jn}}} \qquad \forall i \in C_n \tag{4.6}$$

where the numerator represents the conditional indirect utility for a specific site, i and the denominator is the sum of the conditional indirect utilities over all the alternatives in C_n. Equation (4.6) follows a logistic (logit) probability distribution. The distinction between logit and other forms of random utility is how the error terms are distributed across the alternatives. The extreme value distribution is preferred since it is relatively easy to fit in estimation, approximates the normal distribution very closely and can be easily implemented in the generalized case of more than two alternatives (Deaton and Muellbauer, 1980).

The choice model in (4.6) is known as the multinomial logit (MNL) specification where more than two choices are possible. Multinomial logit models are frequently used in recreational demand where environmental quality is an important determinant of choice. The MNL model is convenient when the choice set or number of attributes is large. This advantage can also be an important disadvantage when there exists a high degree of correlation between site attributes. If sites are closely related through the estimated attribute's error terms, this may alter the probabilities associated with each site choice. This fundamental concern, in the logit framework, is a violation of the Independence of Irrelevant Alternatives (IIA) property. The property states that the introduction or deletion of one alternative from the choice set cannot alter the probability of choosing any of the remaining alternatives.

From the above analysis, it would be of interest to measure the welfare associated with a change in the characteristics of a hunting site. To measure an environmental quality change we must look at the difference in utility before and after the change. Using the indirect utility function defined above, *V*, the compensating variation (CV) is given by:

$$V(P,Q^0,M) = V(P,Q^1,M+CV) \tag{4.7}$$

The condition says that utility will remain constant after the quality change, Q^0 to Q^1, given the increased compensation, *CV*. The estimated coefficients of the indirect utility function can therefore be used to elicit *CV*. The estimated parameters are applied to the choice probabilities for the individual sites in the choice set. Small and Rosen (1981) initially researched welfare measures in discrete choice models and this was extended by Hanemann (1982, 1984). Integrating the estimated coefficients with the definition of CV, the welfare measure used to examine the impact of a quality change is:

$$W = \frac{1}{\mu}\left[\ln\sum_{i=1}^{n} e^{V_{1i}} - \ln\sum_{i=1}^{n} e^{V_{0i}}\right] \tag{4.8}$$

where μ is the marginal utility of income (the coefficient on the travel cost variable), V_{0i} and V_{1i} are the utility level of individual *i* before and after the change. Hanemann (1982) shows that μ is the negative of the coefficient on the travel cost parameter estimated in the random utility models.

Equation 4.6 is used in our model to calculate the probability of hunters visiting any of the locations in the FMA area. Changes in these probabilities show the impact of the harvest schedule on hunting behaviour. Consequently, we can observe how hunters change behaviour as the timber schedule and other forest characteristics change. We also apply Equation (4.8) to assess the changes in the welfare of hunters resulting from changes in the forest characteristics due to harvesting activities on the FMA.

PROBLEM DESCRIPTION AND FORMULATION

The Indirect Utility Function

Before presenting the mathematical formulation of the problem under study, it is important to briefly discuss the indirect utility function that is used to measure the benefits of hunting. The indirect utility, V_{hjt}, is the maximum utility an individual hunter (h) gets by hunting in location j in period t in the study area. This utility is estimated using the linear indirect utility function given in Equation 4.9.

$$V_{hjt} = \beta_0 C_{jh} + \beta_1 z_{jt} + \beta_2 \left(\frac{\sum_{s=-T^0}^{t-1-z} x_{sjt-1} + \sum_{s=-T^0}^{t-z} x_{sjt}}{A_{jt}} \right) + \beta_3 \sum_{s=-T^0}^{t} \sum_{t'=t+1}^{T} x_{st'j} HSI_{jst} + \beta_4 \ln(A_{jt}) \quad (4.9)$$

Where:

$z_{jt} = 0,1$ access variable for location j in period t.

C_{jh} = the travel cost of hunter h to location (site) j, measured in hundreds of dollars

A_{jt} = total *forested* area (in km^2) of location (site) j in period t.

$\sum_{s=-T^0}^{t-1-z} x_{sjt-1}$ = the total area harvested in location j in the previous period (t-1).

$\sum_{s=-T^0}^{t-z} x_{sjt}$ = the total area harvested in location j in period t.

HSI_{jst} = the average Habitat Suitability Index (HSI) of location j at time t for stands born in time s. This HSI comes from the study of Buckmaster *et al.* (1999).

The habitat suitability index of elk is a rating between zero (very poor habitat) and one (very good habitat) of the quality of a site for elk. HSI models predict the suitability of a habitat for species based on an assessment of habitat attributes such as habitat structure, habitat type and spatial arrangements between habitat features (Buckmaster *et al.* 1999). For forested areas of the FMA, this is calculated as $HSI_{ijst} = S_2 * S_3$, where S_2 is the habitat suitability index associated with percent tree canopy closure, and S_3 represents the habitat suitability index associated with per cent deciduous tree canopy cover (percent composition of deciduous tree species in the tree canopy). From the definition of the HSI for the stands, we can obtain the average HSI (weighted by the area of stands) for location j in period t as: $HSI_{jst} = \frac{1}{A_{jt}} \sum_{i=1}^{I} \sum_{s=-T^0}^{t} \sum_{t'=t+1}^{T} x_{ist'j} HSI_{ijst}$. The final measure of a site's quality however, is the elk habitat units (EHU), which is obtained by multiplying the HSI of a stand by its corresponding area. The EHUs for a location is the potential carrying capacity of the site measured in elf per ha. The total number of habitat units depends on the size of the stand. To get the total EHUs for a location, we multiply the average HSI of the location by the total area of that location. The elk habitat units (EHU) for location j in period t is therefore given as: $EHU_{jt} = \sum_{s=-T^0}^{t} \sum_{t'=t+1}^{T} x_{st'j} HSI_{jst}$, which is used in Equation 4.9. Note we are summing up HSIs for stands born before time t but which are also harvested after time t in period t'.

The parameters for the indirect utility function described above were estimated by Haener *et al.* (2001) for the FootHills Model Forest (FMF) using data obtained through a survey of elk hunters from several communities in Alberta including Hinton, Edson, and Calgary. The parameters of the utility function are given in Table 4.1. The table shows that with the exception of access and percentage area harvested, the other three variables are highly significant. The signs on the coefficients of the random utility model in Table 4.1 suggest the likely impacts of hunter preferences on the timber harvest schedule. Hunters are likely to avoid accessed locations that are far away from Drayton Valley and Rocky Mountain House, and locations with large opened forest canopies. They are more likely to prefer locations with large numbers of elk and small canopy openings. This suggests that a likely impact of the hunter benefits in this model will be to concentrate harvests to more remote locations.

Table 4.1. Estimated parameters of the random utility model in the Foot Hills Model Forest.

Variable	Coefficient	Standard error	Probability
Travel Cost (β_0)	-1.1982	0.1451	0.000
Access (β_1)	-0.1043	0.2487	0.3375
% of total area harvested in last 10 years (β_2)	-0.1941	0.1287	0.0658
Elk Habitat Units (β_3)	0.3457	0.0748	0.000
Log (Area) (β_4)	0.6097	0.0937	0.000

Source: Haener et al. (2001)

Model Formulation

To ease comparison of this model with others in the literature, we present the explicit mathematical formulation for the forest management problem. The objective function of the problem to be solved is the net present value from timber sales, utility from hunting and the value of ending inventory less the management, harvests, transportation, and access road costs. The model is an extension of the Model II formulation given in Johnson and Scheurman (1977). It is important to point out that this formulation and solution technique are similar to the Access Model formulation of Chapter 3, except for the addition of the benefits due to elk hunting.

First, we define the following sets and variables:

Let

J be the set of all *supply locations* in the forest with j serving as a counter (j=1,...,J).

J^P be the set of permanently accessed supply locations.

J^l be the set of all permanently accessed locations that are adjacent to at least one location that is not accessed.

I be the set of all *demand locations* with i serving as a counter (i=1,...,I)

I_j^A be the set of all *supply locations* adjacent to j from which product may be shipped. This set is empty for all $j \in J^P - J^1$ (i.e., permanently opened locations that are not adjacent to any unaccessed locations).

I_j^B be the set of all *supply locations* adjacent to j to which product may be shipped.

I_j^C be the set of all *demand locations* to which product may be shipped from j. This set is empty for all $j \notin J^P$.Note that j is only aplicable to permanently accessed locations.

y_{jit}^P be the volume shipped from supply location j to demand location i for $j \in J^P$.

y_{jkt}^A be the volume shipped from supply location j to supply location k for $j,k \notin J^P$ and $j \in J^1$.

y_{jt}^s be the volume supply at location j.

y_{it}^d be the volume demand at demand location i.

z_{jt} be the 0,1 access variable for location j.

δ = an arbitrary weight on the non-timber benefits

The objective function is given in Equation 4.10 as:

$$\max \delta \sum_{t=1}^{T} \beta^{t-1} \sum_{h=1}^{H} \frac{n_h}{\beta_0} \ln \sum_{j=1}^{J} \exp \left(\begin{array}{l} \beta_0 C_{jh} + \beta_1 z_{jt} + \beta_2 \left(\dfrac{\sum\limits_{s=-T^0}^{t-1-z} x_{sjt-1} + \sum\limits_{s=-T^0}^{t-z} x_{sjt}}{A_{jt}} \right) \\ + \beta_3 \sum\limits_{s=-T^0}^{t} \sum\limits_{t'=t+1}^{T} x_{st'j} HSI_{jst} + \beta_4 \ln(A_{jt}) \end{array} \right) + \sum_{i}^{I} \sum_{t}^{T} R(y_{it}^d)$$

$$+ \sum_t \sum_j E_{jt} w_{jt} - \sum_{s=-T^0}^{t-z} \sum_{s \geq s+z}^{T} \sum_{j}^{J} c_{stj} x_{stj} - \sum_t \sum_{j \in J^P} c_{jt}^A z_{jt} - \sum_t \sum_{j \in J^P} \sum_{k \in I_j^A} c_{kjt}^s y_{kjt}^A$$

$$- \sum_t \sum_{j \in J^P} \sum_i c_{jit}^s y_{jit}^P \tag{4.10}$$

subject to:

$$y_{jt}^s \leq \sum_{s=-T^0}^{t-z} v_{sjt} x_{sjt} + \sum_{k \in I_j^A} y_{kjt}^A \qquad \forall j, t \; (I_j^A = \phi \text{ for } j \in J^P - J^1) \tag{4.11}$$

$$\sum_{k \in I_j^B} y_{jkt}^A \le y_{jt}^s \qquad \forall j \notin J^p \tag{4.12}$$

$$\sum_{i \in I_j^t} y_{jit}^p \le y_{it}^s \qquad \forall j \in J^p \tag{4.13}$$

$$y_{jt}^s \le y_{jt}^s z_{jt} \qquad \forall j \notin J^p \tag{4.14}$$

$$\sum_{s=-T^0}^{t-z} x_{sjt} = \sum_{k=t+z}^{T} x_{tkj} + w_{jt} \qquad \forall j, t = 0, \ldots, T \tag{4.15}$$

$$\sum_{t=s+z}^{T} x_{sjt} + w_{jt} = A_{sj} \qquad \forall j, s = -T^0, \ldots, 0 \tag{4.16}$$

$$y_{it}^d \le \sum_{i \in I_j^t} y_{jit}^p \qquad \forall i, t \tag{4.17}$$

$$x_{sjt} \ge 0 \qquad \forall sjt$$

$$w_{sj} \ge 0 \qquad \forall sj$$

$$y_{it}^d \ge 0 \qquad \forall t$$

The remaining variables used are defined as:

$R(y_{it}^d)$ = the revenue for wood products at demand center *i* in period *t*.

E_{sj} = the discounted value per unit area of managing stand type *j* starting in period *s* and leaving the stand type as ending inventory

w_{jt} = area managed of stand type *j* in period *t* and left as ending inventory

x_{sjt} = area managed on stand type *j* in period *s* and final harvest in period *t*

The parameters are defined as:

c_{jt}^A = discounted cost of accessing (road construction) location *j* in period *t*

c_{kjt}^s = discounted cost/m^3 of shipping wood from location *k* to *j* in period *t*

c_{jit}^s = discounted cost/m^3 of shipping wood from permanently accessed location *j* to demand center *i* in period *t*

A_{sj} = the number of area unit of stand type *j* in the first period that were regenerated in period *s*.

c_{sjt} = the discounted cost per unit area of managing stand type j starting in period s and final harvest in period t

v_{sjt} = the merchantable volume per unit in period t, when stand type j is regenerated in period s.

z = minimum time between regeneration and harvest

T = the number of planning periods in the planning horizon

T^0 = number of periods before period zero in which the oldest age class present in period one was regenerated.

n_h = number of hunters of type h visiting sites in the study area

β = discount factor (using a 5% discount rate)

Although the above mixed integer non-linear programming model can be easily recognized as similar to the well known Model II structure given in Johnson and Scheurman (1977), the interpretations provided below are aimed at clarifying its meaning.

Equation (4.10) is the objective function. The purpose of our model is to maximize the net present value of both timber and non-timber values for the FMA over the 100-year planning horizon. The first term in the objective function is the discounted benefits of elk hunting in the FMA. These hunting benefits are estimated using the indirect utility function given in Equation (4.9) and converted to dollar terms by dividing the utility by the marginal utility of income (β_0). The second term is the discounted revenue from the sale of final wood products (lumber and oriented strand board [OSB]), whilst the third term is the value of ending inventory. From these benefits, we subtract the cost of regeneration, harvesting, road construction, costs of shipping wood from one location to an adjacent location and the cost of shipping wood from permanently accessed locations to the demand center (mills). The weighting factor (δ) is an arbitrary weight that can be used to adjust the weight on the non-timber values, given that the timber values would most likely overshadow the hunting values.

Equation set (4.11) says that the volume supply of wood in a given location is the sum of wood supplied from that location plus any wood that is shipped through that location in any time period. Equation (4.12) accounts for all wood in non-permanently accessed locations. Specifically, it states that the volume of wood shipped from one location to another location cannot be greater than the volume supply of wood in the initial location. Equation (4.13) is applicable to only locations with permanent access, and implies that the volume of wood shipped from a permanently accessed location to a mill cannot be greater than the volume supply of wood in the supply location. The model must be further constrained to ensure that each stand is accessible when it is to be harvested. This is achieved with Equation (4.14). Equation (4.14) is applicable to non-permanently accessed locations, and says that wood cannot flow from an unaccessed location. Particular note should be taken of Equation (4.14) because it is quite different from the rest in that it is a non-linear constraint in which the access variable (z_{jt}) is a binary integer variable. Equation (4.15) accounts for area regenerated during the planning period. Total area harvested during the planning period plus area left as ending inventory at the end of the planning period should equal area regenerated during the planning period. This constraint ensures that all harvested areas are regenerated. Equation (4.16) accounts for the forest area regenerated before the planning period (existing stands). Total area harvested during the planning horizon plus area left as ending inventory (at the end of the planning horizon) should equal the initial area (regenerated in period s before planning period). Equation (4.17) is the mill demand constraints. These ensure that the wood shipped from permanently accessed locations equal the mill demands.

The forest-scheduling problem as presented above will be difficult to solve using traditional linear programming solution techniques due to its large size (approximately 2.6 million decision variables and 96 thousand constraints). The simulation approach of Hoganson and Rose (1984) that we have adopted enables us to develop an algorithm to solve the forest management problem by decomposing it into a series of smaller, easily solved problems. This solution technique relies on a direct interpretation of the dual variables that result from the lagrangian formed using Equations 4.10 to 4.17. Even if the above problem could be solved directly, we are still motivated to use the decomposition technique because the dual variables we derive provide additional insights and relationships to forest rotation models in the literature, especially, the Hartman (1976) optimal forest rotation model. Furthermore, due to the considerable uncertainties associated with many of the key parameters (e.g., prices, mill demands, yields) in long-term planning models, it is worthwhile to relax the feasibility requirements. This relaxation is central to our solution technique, but is not easily achieved in standard linear programming solution approaches.

The lagrangian to the primal problem is given as:

$$
\begin{aligned}
L = {} & \delta \sum_{t=1}^{T} \beta^{t-1} \sum_{h=1}^{H} \frac{n_h}{\beta_0} \ln \sum_{j=1}^{J} \exp \left(\begin{aligned} & \beta_0 C_{jh} + \beta_1 z_{jt} + \beta_2 \left(\frac{\sum_{s=-T^0}^{t-1-z} x_{sjt-1} + \sum_{s=-T^0}^{t-z} x_{sjt}}{A_{jt}} \right) \\ & + \beta_3 \sum_{s=-T^0}^{t} \sum_{t'=t+1}^{T} x_{st'j} HSI_{jst} + \beta_4 \ln(A_{jt}) \end{aligned} \right) \\
& + \sum_i \sum_t R(y_{it}^d) - \sum_{s=-T^0}^{t-z} \sum_{s \ge s+z}^{T} \sum_j^{J} c_{stj} x_{stj} - \sum_t \sum_{j \in J^P} c_{jt}^A z_{jt} - \sum_t \sum_{j \in J^P} \sum_{k \in I_j^A} c_{kjt}^s y_{kjt}^A - \sum_t \sum_{j \in J^P} \sum_i c_{jit}^s y_{jit}^p \\
& + \sum_j \sum_t \pi_{it} \left(\sum_{i \in I_j^c} y_{jit}^p - y_{it}^d \right) + \sum_j \sum_t u_{jt} \left(\sum_{s=-T^0}^{t-z} v_{sjt} x_{sjt} + \sum_{k \in I_j^A} y_{kjt}^A - y_{jt}^s \right) + \sum_{j \in J^P} \sum_t v_{jt} \left(y_{jt}^s - \sum_{k \in I_j^B} y_{kjt}^A \right) \\
& + \sum_{j \in J^P} \sum_t \theta_{jt} \left(y_{it}^s - \sum_{i \in I_j^c} y_{jit}^p \right) + \sum_{j \in J^p} \sum_t \lambda_{jt} \left(y_{jt}^s z_{jt} - y_{jt}^s \right) + \sum_j \sum_{t=0}^{T} s_{jt} \left(\sum_{s=-T^0}^{t-z} x_{sjt} - \sum_{k=t+m}^{T} x_{tkj} + w_{jt} \right) \\
& + \sum_j \sum_{s=-T^0}^{0} a_{sj} \left(A_{sj} - \sum_{t=s+z}^{T} x_{sjt} + w_{jt} \right) \qquad (4.18)
\end{aligned}
$$

First order conditions for continuous variables in the model are:

Let

$$
e^{X_{hjt}\mathrm{B}} = \exp \left(\beta_0 C_{jh} + \beta_1 z_{jt} + \beta_2 \left(\frac{\sum_{s=-T^0}^{t-1-z} x_{sjt-1} + \sum_{s=-T^0}^{t-z} x_{sjt}}{A_{jt}} \right) + \beta_3 \sum_{s=-T^0}^{t} \sum_{t'=t+1}^{T} x_{st'j} HSI_{jst} + \beta_4 \ln(A_{jt}) \right)
$$

$$
L_{y_{it}^d} = R'(y_{it}^d) - \pi_{it} = 0 \qquad (4.19)
$$

$$L_{x_{sjt}} = -c_{sjt} + u_{jt}v_{sjt} - a_{sj} + s_{jt} + \frac{\delta\beta^{t-1}}{\beta_0}\sum_{h=1}^{H} n_h\left(\sum_{t'=s}^{t-1}\left(\beta_3 HSI_{jst'}\frac{e^{X_{hjt}\cdot\mathrm{B}}}{\sum_{k=1}^{J} e^{X_{kt}\cdot\mathrm{B}}}\right) + \sum_{t'=t}^{t+1}\left(\frac{\beta_2}{A_{jt}}\frac{e^{X_{hjt}\cdot\mathrm{B}}}{\sum_{k=1}^{J} e^{X_t\cdot\mathrm{B}}}\right)\right) \le 0,$$

$$= 0 \; if \; x_{sjt} > 0 \quad \forall\, j, s = -T,\ldots,0 \; and \; t = 0,\ldots,T \qquad (4.20)$$

$$L_{x_{sjt}} = -c_{sjt} + u_{jt}v_{sjt} - s_{sj} + s_{jt} + \frac{\delta\beta^{t-1}}{\beta_0}\sum_{h=1}^{H} n_h\left(\sum_{t'=s}^{t-1}\left(\beta_3 HSI_{jst'}\frac{e^{X_{hjt}\cdot\mathrm{B}}}{\sum_{k=1}^{J} e^{X_{kt}\cdot\mathrm{B}}}\right) + \sum_{t'=t}^{t+1}\left(\frac{\beta_2}{A_{jt}}\frac{e^{X_{hjt}\cdot\mathrm{B}}}{\sum_{k=1}^{J} e^{X_t\cdot\mathrm{B}}}\right)\right) \le 0,$$

$$= 0 \; if \; x_{sjt} > 0 \quad \forall\, j, s = 0,\ldots,T \; and \; t = s + z,\ldots,T \qquad (4.21)$$

$$L_{y^A_{kjt}} = -c^s_{kjt} + u_{jt} - v_{kt} \le 0, = 0 \;\; if \;\; y^A_{kjt} > 0 \qquad \forall k \notin J^p, j \in I^B_k, t \qquad (4.22)$$

$$L_{y^A_{kjt}} = -c^s_{jit} + \theta^s_{jt} + \pi_{it} \le 0, = 0 \;\; if \;\; y^p_{jit} > 0 \qquad \forall j \in J^p, i, t \qquad (4.23)$$

$$L_{y^s_{jt}} = -u_{jt} + v_{jt} + \lambda_{jt}(z_{jt} - 1) \le 0, = 0 \;\; if \;\; y^s_{jt} > 0 \qquad \forall j \notin J^p, t \qquad (4.24)$$

$$L_{y^s_{jt}} = -u_{jt} + \theta_{jt} \le 0, = 0 \;\; if \;\; y^s_{jt} > 0 \qquad \forall j \in J^p, t \qquad (4.25)$$

$$L_{w_{jt}} = E_{jt} - a_{sj} \le 0; = 0 \; if \quad w_{jt} > 0 \qquad \forall s = -T^0,\ldots\ldots,0 \qquad (4.26)$$

$$L_{w_{jt}} = E_{jt} - s_{sj} \le 0; = 0 \; if \quad w_{jt} > 0 \qquad \forall s = 0,\ldots\ldots,T \qquad (4.27)$$

The first order condition for the integer variables z_{jt} are handled differently from the rest because the Lagrangian function is not differentiable with respect to z_{jt}. (since z_{jt} is a 0, 1 variable). Hence, we work with the difference in the utility function over $z_{jt} = 1$ and $z_{jt} = 0$.

Let

$$e^{(z,X)_{jt}\mathrm{B}} = \exp\left(\beta_0 C_{jh} + \beta_1 z_{jt} + \beta_2\left(\frac{\sum_{s=-T^0}^{t-1-z} x_{sjt-1} + \sum_{s=-T^0}^{t-z} x_{sjt}}{A_{jt}}\right) + \beta_3 \sum_{s=-T^0}^{t}\sum_{t'=t+1}^{T} x_{st'j} HSI_{jst} + \beta_4 \ln(A_{jt})\right)$$

$$L_{z=1} - L_{z=0} = -c_{jt}^{A} + \lambda_{jt} y_{jt}^{s} + \delta \frac{\beta^{t-1}}{\beta_0} \sum_{h}^{H} n_h \left(\beta_1 \log e^{(1,X)_{hjt}\mathrm{B}} - \beta_1 \log e^{(0,X)_{hjt}\mathrm{B}} \right) \leq 0, \tag{4.28}$$

$$= 0 \;\; if \;\; z_{jt} = 1$$

Economic Interpretation of the First Order Conditions

In this section the first order conditions of the lagrangian function are interpreted from an economic perspective. Many of the first order conditions show the linkage between the forest level model in Equations 4.10 to 4.17 to single stand forest rotation models. In addition, the interpretations show how the spatial constraints implied by the forest access and the spatial choice model for hunters are translated into stand level shadow prices, which makes the stand level problems easy to solve.

Equation (4.19) can be re-arranged to give Equation (4.29), which means that marginal revenue equals the price of the wood product.

$$R'(y_{it}^{d}) = \pi_{it} \tag{4.29}$$

Equation 4.30 is obtained by re-arranging Equation 4.20. The right hand side of Equation 4.30 is the timber value of the wood at rotation minus the cost of growing plus the value of the next rotation (s_{jt}) for every rotation t = $s+z,\ldots,T$, plus the marginal increase in the recreational hunting value over the rotation period t-s.

$$a_{sj} \geq -c_{sjt} + u_{jt} v_{sjt} + s_{jt} + \frac{\delta}{\beta_0} \sum_{h=1}^{H} n_h \left(\sum_{t'=s}^{t-1} \left(\beta^{t'-1} \beta_2 HSI_{jst'} \frac{e^{X_{hjt'} \cdot \mathrm{B}}}{\sum_{k=1}^{J} e^{X_{hkt'} \cdot \mathrm{B}}} \right) + \sum_{t'=t}^{t+1} \left(\frac{\beta^{t'-1} \beta_3}{A_{jt}} \frac{e^{X_{hjt'} \cdot \mathrm{B}}}{\sum_{k=1}^{J} e^{X_{hjt'} \cdot \mathrm{B}}} \right) \right)$$

$$if \;\; x_{sjt} > 0 \quad \forall\, j, s = -T,\ldots,0 \; and \; t = 0,\ldots, Max\ Rotation \tag{4.30}$$

Equation 4.30 implies that the dual variable a_{sj}, which is interpreted as the land value of type j if born in period s, is bounded from below by the expression on the right for every t from $s+z$ to T. This means the land value (a_{jt}) is equal to the rotation t that gives the maximum value of combined timber and non-timber benefits. This first order condition is similar to the Hartman forest rotation model for a single stand (Hartman, 1976) since we maximize the flow of discounted recreational values of the forest while the forest is standing plus the discounted value of the forest when the trees are harvested. Therefore the optimal harvest period occurs where both timber and recreational values are maximized. It should however, be noted that the optimal timing of harvests in this model is modified from the Hartman (1976) model by the presence of the access constraint (Equation 4.14). This equation modifies the prices of timber products to shift the harvest cycle away from certain periods where there are few adjacent stands (i.e. stands within the same location) being harvested.

The first order condition 4.21 is similar to 4.20 and can be re-arranged as:

$$s_{js} \geq -c_{sjt} + u_{jt} v_{sjt} + s_{jt} + \frac{\delta}{\beta_0} \sum_{h=1}^{H} n_h \left(\sum_{t'=s}^{t-1} \left(\beta^{t'-1} \beta_2 HSI_{jst'} \frac{e^{X_{hjt'} \cdot \mathrm{B}}}{\sum_{k=1}^{J} e^{X_{hkt} \cdot \mathrm{B}}} \right) + \sum_{t'=t}^{t+1} \left(\beta^{t'-1} \frac{\beta_3}{A_{jt}} \frac{e^{X_{hjt} \cdot \mathrm{B}}}{\sum_{k=1}^{J} e^{X_{hjt} \cdot \mathrm{B}}} \right) \right)$$

$$\textit{if } x_{sjt} > 0 \quad \forall\, j, s = 0, \ldots, T \textit{ and } t = s + z, \ldots, \textit{Max Rotation} \qquad (4.31)$$

The dual variable s_{sj}, again represents the bare land value for type j born in period s. The interpretation of this equation is the same as for Equation 4.30.

Rearranging Equation 4.28 gives Equation 4.32, which relates the timber and non-timber benefits on the left hand side to the cost of providing access to location j in period t on the right hand side. This equation determines the conditions under which locations should be opened or closed.

$$\lambda_{jt} y_{jt}^{s} + \delta \frac{\beta^{t-1}}{\beta_0} \sum_{h}^{H} n_h \left(\beta_1 \log e^{(1,X)_{hjt} \mathrm{B}} - \beta_1 \log e^{(0,X)_{hjt} \mathrm{B}} \right) \geq c_{jt}^{A} \quad \textit{if } z_{jt} = 1 \qquad (4.32)$$

The equation basically states that the value of wood crossing over the location j in period t and the increase in hunting value due to opening up the location should be greater than or equal to the access cost. If the left hand side of Equation 4.32 is greater than the right hand side, then the location should be opened, otherwise the location is closed. This equation also provides some insights into the likely impacts of the non-timber benefits on access road development. From Table 4.1, we notice that the sign on the access coefficient (β_l) is negative, and so an area with large hunting benefits everything else equal will tend to stay closed. The overall effect on the schedule should be to concentrate harvests compared to a situation without non-timber values.

The remaining first order conditions (4.22 to 4.27) are interpreted the same way as in Chapter 3. Therefore these conditions are only briefly mentioned in this chapter. Re-arranging Equations 4.22 to 4.24 give the first order conditions (4.33 to 4.36):

$$v_{kt} = u_{jt} - c^{s}_{kjt} \;\; \text{if} \;\; y^{A}_{kjt} > 0 \;\text{otherwise}\; v_{kt} > u_{jt} - c^{s}_{kjt} \;\; \Rightarrow y^{A}_{kjt} = 0 \;\; \forall k \notin J^{P}, j \in I^{B}_{k}, t \tag{4.33}$$

$$\theta^{s}_{jt} \geq \pi_{it} - c^{s}_{jit} = 0 \;\; \text{if} \;\; y^{P}_{jit} > 0 \text{ over all } i. \text{ For each } j \in J^{P} \text{and } t \tag{4.34}$$

$$u_{jt} \geq v_{jt} + \lambda_{jt}(z_{jt} - 1) = 0 \;\; \text{if } y^{s}_{jt} > 0 \text{ for all } j \notin J^{P}, t. \tag{4.35}$$

$$\theta^{s}_{jt} \leq u_{jt}, \; = \; 0 \;\; \text{if } y^{s}_{jt} > 0 \text{ for all } j \in J^{P}, t. \tag{4.36}$$

Equation 4.33 says that the net value of wood at location *k* is equal to the maximum value over all shipping alternatives from *k*. This implies the wood should be moved to the location that gives the highest net value. Equation 4.34 implies that the value of wood at location *j* is equal to the mill price minus transportation costs. Equation 4.35 implies that the net value of wood at *j* is equal to the value of wood at *j* minus the access cost adjustment. Equation 4.36 says that for permanently accessed locations, $\theta^{s}_{jt} = u_{jt}$. Note that θ^{s}_{jt} is the net price of wood in permanently accessed locations and u_{jt} is the price of wood in non-permanently accessed locations.

Equation 4.37 is a combination of Equations 4.23 and 4.27.

$$u_{jt} = \max_{k \in I^{B}_{j}} \{u_{kt} - c^{s}_{jkt}\} + \lambda_{jt}(z_{jt} - 1) \tag{4.37}$$

This is solved using dynamic programming in two stages. The details of the solution procedure are given in Appendix IV.

$$a_{sj} \geq E_{jt} \tag{4.38}$$

$$s_{sj} \geq E_{jt} \tag{4.39}$$

Equation (4.38) means the bareland value of the existing stand for any analysis area is at least as great as its value if left as ending inventory. The meaning of Equation (4.39) is that the bareland value of the regenerated stand for any analysis area and is at least as great as its value if left as ending inventory. Note that Equations 4.38 and 4.39 include the sum of non-timber benefits to the end of the planning horizon.

Solution Procedure

The size of the current problem is very similar to the Access Model described in Chapter 3. The number of constraints is the same (96,000), including 3,850 integer constraints. Because of the similarities between this and the Access Model in Chapter 3, the solution of the stand level management problem, as well as the shadow price adjustments for the access constraints are identical to what has already been described in Chapter 3. The emphasis in this section will be to describe how the inclusion of the random utility model affects the solution procedure. In the previous Access Model, the optimal rotation age was determined based only on the timber values. However, inclusion of non-timber values is expected to influence the optimal rotation ages, and the criterion for determining which location should be opened or not. This means that optimal rotation occurs where both timber and non-timber benefits are maximized. In addition, the decision to open a location is now based on whether the total value of timber and non-timber benefits is greater than the costs of doing so. Due to the similarity between the solution procedure and the Access Model, the procedure is not repeated here, but a detailed algorithm is provided in Appendix V.

Model Scenarios

In order to investigate the impacts of timber harvesting on the non-timber values, as well as the effect of including non-timber benefits on harvest scheduling, we examined six different scenarios of the model specified above. Each model run is comprised of combinations of different levels of timber volumes harvested, average number of hunters visiting the study area, and weight (δ) on the hunter benefits specified in Table 4.2. The third column shows the average number of hunters that visit a location in any given year. The hunters visit the study area either from Drayton Valley (DV) or Rocky Mountain House (RMH). The number of hunters was estimated based on the sales of elk hunting licences for the study area from the Fish and Wildlife Division of Alberta Environment (Alberta Environment, 2001). The fourth column shows an estimate of the maximum price in terms of $\$/m^3$ of final product that could be paid at the mill gate. The last two columns give the maximum volume of final products that can be consumed by each mill on an annual basis. In this set of runs demands for hardwoods and softwoods add up approximately to the total volumes of these species harvested on the Drayton Valley FMA area during the last 5 years. Hence, the maximum volumes harvestable in each area represent a combination of mill capacities and allowable cuts.

Baserun 1 includes the timber harvest levels and number of hunters specified in Table 4.2, with δ set to zero. The purpose of this run is to estimate the timber values and determine the harvest schedule when hunter preferences are not included in the objective function of the model. By comparing the results from Baserun 1 with Scenario 1 (which contains both timber and non-timber values), we can determine the effect of including non-timber benefits on timber values, the harvest schedule, and access road development. Secondly, the difference between Baserun 1 and Baserun 2 will illustrate the overall impact on non-timber values of timber harvesting without considering the non-timber values in the objective function.

Baserun 2 was specified to estimate the value of recreational hunting when no timber harvests takes place. Therefore the model run contains zero harvest levels with a 1000 hunters. The idea is to find out what the value of non-timber benefits are when there is no timber harvesting in the forests. We then use this as a basis for determining the effect of harvesting on non-timber values by comparing this run with Scenario 1. This comparison also allows us to examine a) whether timber harvesting increase or decrease hunter welfare, and b) how timber harvesting changes the distribution of hunters on the landscape.

Scenario 1 and 2 both comprise of a 1000 hunters and the same level of wood volumes given in Table 4.2. These two scenarios were designed to test the effect of increasing the weight on the non-timber values on the timber values and harvest schedule, whilst keeping the level of timber harvests the same as in Scenario 1.

Scenario 3 has 1000 hunters visiting the FMA and δ set to zero, with a timber harvest level reduced by 20% relative to Scenario 1. Scenario 4 has the same timber harvest level as Scenario 3, but with δ set to one. Scenarios 3 and 4 were designed to examine the impacts of reduced harvesting on non-timber values, and the distribution of hunters on the landscape. We can also evaluate the effect of non-timber values on timber values and the harvest schedule at lower timber harvest levels. This information will help us understand whether the impacts of the non-timber values on timber values and harvest schedule are similar at high and low mill demands. In other words, is it possible to eliminate or change the direction of the effects of non-timber values on timber values if we reduced the level of harvests by 20%?

Table 4.2. Summary of the average number of hunters, maximum price of final product and maximum mill demands for the six model scenarios investigated.

Model run	δ	Average number of Hunters/yr	Max price/m^3 of final product (Lumber /OSB)	Max. Mill Demand (m^3/yr)	
				Lumber	OSB
Baserun 1	0	1000	300/100	90,000	200,000
Baserun 2	1	1000	0/0	0	0
Scenario 1	1	1000	300/100	90,000	200,000
Scenario 2	5	1000	300/100	90,000	200,000
Scenario 3	0	1000	300/100	72,000	160,000
Scenario 4	1	1000	300/100	72,000	160,000

RESULTS

Model performance

The six model scenarios described in Table 4.2 were run on a microcomputer with a Pentium 500 Mhz microprocessor, using initial guesses at the marginal costs for both lumber and OSB. The criteria for determining when to stop a run were based on convergence of the deviations from demand constraints and the objective function. The model takes about 10 minutes (approximately 200 iterations) to arrive at a solution. The maximum deviation of each end product from target demands for any period for the model to converge was set at 3%. Table 4.3 presents the performance of the six model runs in terms of the average absolute deviations of the demand constraints. All models performed well in terms of absolute deviations from the mill demands. All average absolute deviations were less than 3% from target mill demands, and none of the access constraints were violated. The differences between the objective function and lagrangian values, which give indications of model convergence, are given in the last column of Table 4.3.

Table 4.3. Comparisons of the deviations from mill demands, and differences between objective function and Lagrangian values.

Model run	Average absolute deviations (%) from Demand Constraint		Percentage difference between objective function and lagrangian values (%)
	Sawmill	OSB mill	
Baserun 1	1.018	1.852	0.001
Scenario 1	1.575	1.255	0.001
Scenario 2	0.783	2.493	0.003
Scenario 3	0.523	1.678	0.001
Scenario 4	0.442	1.681	0.044

Impact of Non-timber Benefits on Timber Values

One of the main objectives of this paper is to examine the effect of including non-timber benefits into the timber-scheduling model on timber values. Table 4.4 shows the net present values (NPVs) for timber and non-timber benefits and the economic rents from timber harvests for the six model runs. The benefits from timber harvests constituted the major part of the total NPV, whilst non-timber benefits were a small component. The effect of the hunting benefits on timber values is determined by comparing the NPVs of Baserun 1 and Scenario 1. The presence of non-timber benefits reduced timber values when a thousand hunters visited the FMA. When the weight on the non-timber benefits was increased from 1 to 5, there was a further reduction in the timber values. This result is further confirmed by comparing the NPVs of timber for Scenario 3 and 4, which shows that even at lower levels of harvests, there are tradeoffs between non-timber benefits and timber values. The main conclusion from Table 4.4 is that the relationship between timber and non-timber values is competitive, with non-timber values increasing at the expense of timber values when greater weight is attached to non-timber values in the objective function. The slight decrease in timber values when non-timber benefits are included may be due to the increase in marginal costs of producing timber in the presence of non-timber benefits. The presence of hunter benefits in the model ensures that locations with high hunter welfare are not opened for harvesting. Hence the increased marginal costs may be a reflection of the extra costs of finding alternative locations to harvest timber, which may involve road construction, high transportation costs, and less valuable timber.

Table 4.4. Comparison of net present values of timber and non-timber values and the economic rent from timber harvests.

	Net present values (10^6 $)			Economic rent of
Model run	Timber	Non-timber	Total	timber (10^6 $)
Baserun 1	497.8111	0.5125	498.3236	41.59
Baserun 2	0.00	0.5482	0.5482	0.00
Scenario 1	497.8021	0.5325	498.3346	40.59
Scenario 2	497.8014	0.5325*	498.3339	40.98
Scenario 3	399.3005	0.5132	399.8137	20.88
Scenario 4	399.2915	0.5281	399.8196	19.93

*Note: In Scenario 2, the weight (δ) on non-timber values is 5, and so the total non-timber values with the weight is $2.66m.

The information in Table 4.4 can also be used to examine the impacts of harvesting timber on the non-timber values. The overall impact on non-timber values of timber harvesting when non-timber values are not considered in the objective function is $0.51m, which is similar in magnitude to that when the non-timber values are included in the objective function of $0.53 million. The welfare of hunters is inversely related to the volume of timber harvested off the FMA. This is evidenced by the fact that Baserun 2 (zero harvest) has the highest welfare, followed by Scenario 4 (low harvest) and then Scenario 1 (high harvest). This suggests that hunters avoid sites with forestry activities, thereby having to travel to sites that are far away from Drayton Valley and Rocky Mountain House, which reduces their welfare. We expected that the reduction in harvests in Scenario 4 should increase the non-timber values significantly because the reduced harvesting decreases the number of locations accessed and total area harvested. The reason this did not happen may be that the hunters have good substitute sites over the landscape.

The economic rent for each scenario in Table 4.4 was used to compare timber and non-timber values since the objective function values are based on the maximum price levels for the two final products. The economic rents would give a more appropriate comparison of the tradeoffs between timber and non-timber values than the objective function values. The economic rent was calculated as the marginal costs (given in Figures 4.1 and 4.2) minus the average cost of each stand. The results show that increasing non-timber values decreases the economic rents of timber production. As the figures of the marginal costs show, inclusion of non-timber benefits increases the marginal costs of producing timber products, and hence decreases timber rents. The economic rent for Scenario 2 is slightly larger than Scenario 1, although Scenario 2 has a larger weight on non-timber values than Scenario 1. The reason is because Scenario 2 has higher marginal costs compared to Scenario 1, and the same average costs. Therefore, the results of the impacts of recreational hunting on timber values based on the economic rents from timber production are consistent with those obtained using the NPVs.

Figures 4.1 and 4.2 show the marginal costs of production of lumber and OSB at the sawmill and OSB mill respectively. In Figure 4.1, we note the marginal costs are increasing over time for all scenarios. This signals scarcity of timber over the planning horizon. The results show that models with non-timber benefits have consistently higher marginal costs than their corresponding models without non-timber benefits. We conclude that although inclusion of non-timber values in the model does not result in a substantial impact on overall timber values, marginal costs are greater for both lumber and OSB. The main reason for higher marginal costs when non-timber benefits are included is related to the availability of alternative harvest locations for timber that do not negatively impact on non-timber values. The fewer these alternative locations, the higher the marginal costs would be.

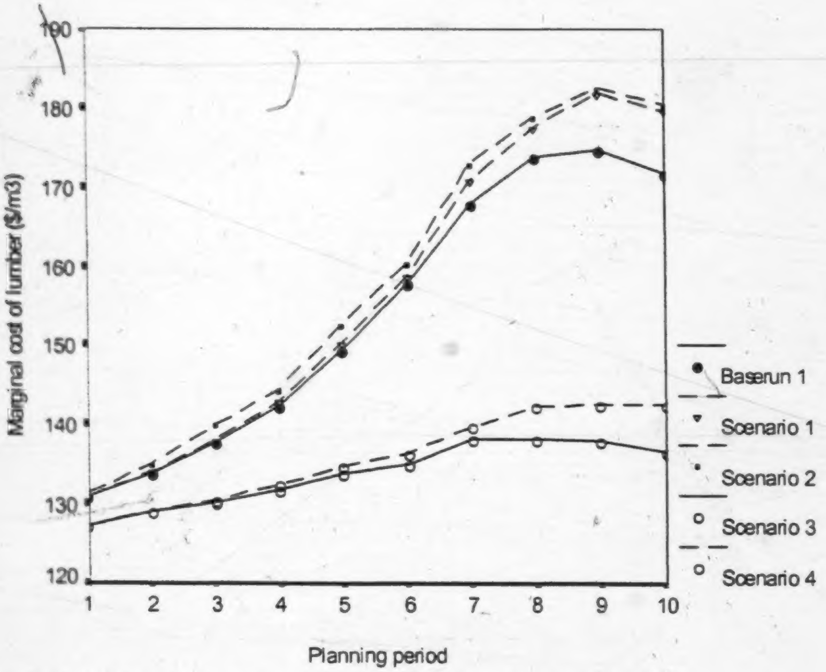

Figure 4.1. Marginal costs of lumber for the planning horizon.

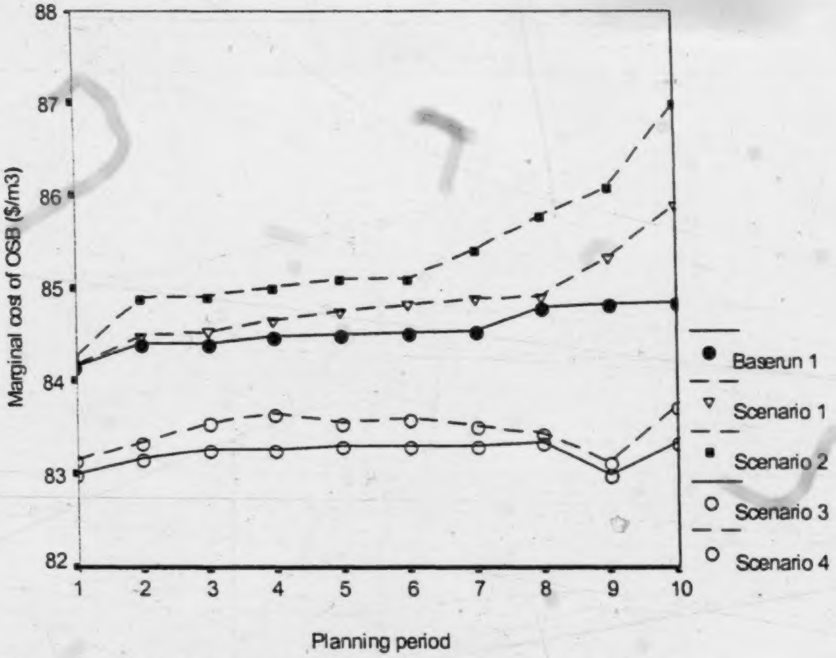

Figure 4.2. Marginal costs of OSB for the planning horizon.

Impact of Hunter Preferences on Harvest Schedule

Thus far, we have examined the impact of including non-timber benefits on timber values. In order to put these effects into better perspective, we examine the influence of hunter preferences for hunting sites in the FMA area on the harvest schedule. As alluded to earlier, the explicit consideration of non-timber benefits in timber supply models affects not only the value of the timber, but also when, where, and how much timber should be harvested in each location. The schedules for the first and last periods in the planning horizon for five of the six scenarios are shown in Figures 4.3 - 4.7. In all models, harvesting spreads out over time, from the first to the last planning period.

Table 4.5 shows that over the planning horizon, the impact of non-timber values on the schedule was a slight concentration of harvests compared to the model without non-timber benefits. This means that on average, more area was harvested within accessed locations when non-timber values were included. We expected that non-timber benefits would significantly reduce the number of locations accessed due to the negative impact of access on hunter utility. A careful analysis of the impact of non-timber values on access road development shows that most of the locations without permanent access that were opened for harvesting had very low non-timber values. Most of these sites had high percentages of areas harvested and low habitat suitability indexes, and consequently, hunters were not attracted to visit such sites. This suggests that the effect of hunter utility on the timber schedule is highly dependent on the magnitude of hunter welfare in each site. These results clearly show that sites with high hunter welfare were left unaccessed, whilst those with low welfare were opened up for timber harvesting.

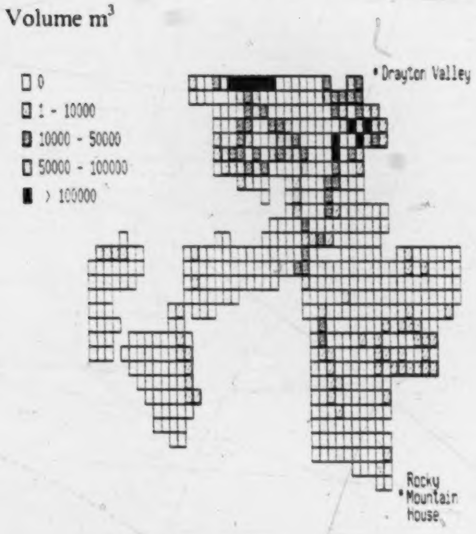
Volume m³
0
1 - 10000
10000 - 50000
50000 - 100000
> 100000
Drayton Valley
Rocky Mountain House

Figure 4.3a. Wood procurement zone for Baserun1 in Period 1. (Number of locations harvested = 144).

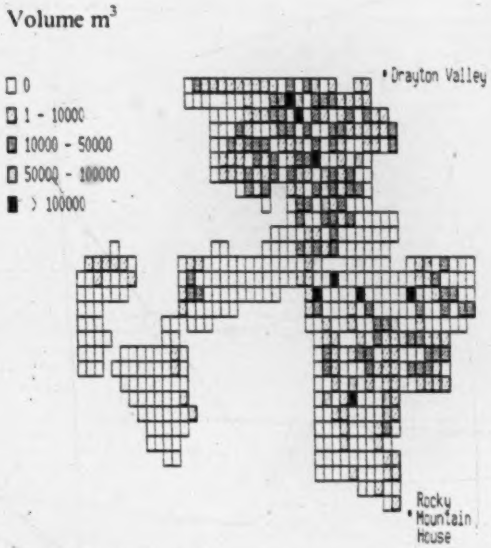
Volume m³
0
1 - 10000
10000 - 50000
50000 - 100000
> 100000
Drayton Valley
Rocky Mountain House

Figure 4.3b. Wood procurement zone for Baserun1 in Period 10. (Number of locations harvested = 238).

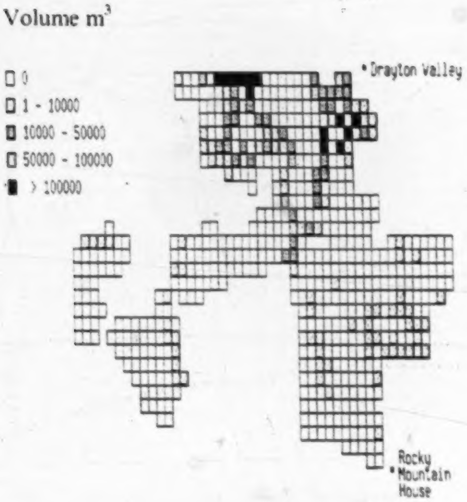
Volume m³
☐ 0
☐ 1 - 10000
▨ 10000 - 50000
☐ 50000 - 100000
■ > 100000
• Drayton Valley
• Rocky Mountain House

Figure 4.4a. Wood procurement zone for Scenario 1 in Period 1. (Number of locations harvested = 144).

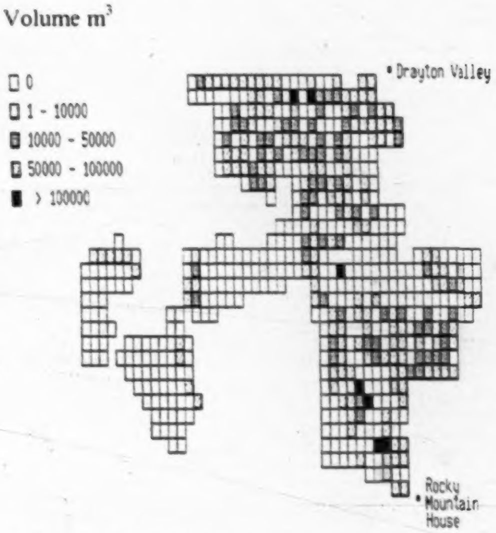
Volume m³
☐ 0
☐ 1 - 10000
▨ 10000 - 50000
☐ 50000 - 100000
■ > 100000
• Drayton Valley
• Rocky Mountain House

Figure 4.4b. Wood procurement zone for Scenario 1 in Period 10. (Number of locations harvested = 233).

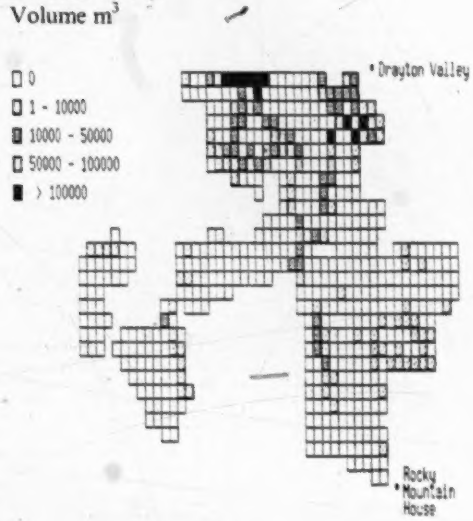
Volume m³
☐ 0
☐ 1 - 10000
▨ 10000 - 50000
☐ 50000 - 100000
■ > 100000
• Drayton Valley
• Rocky Mountain House

Figure 4.5a. Wood procurement zone for Scenario 2 in Period 1. (Number of locations harvested = 144).

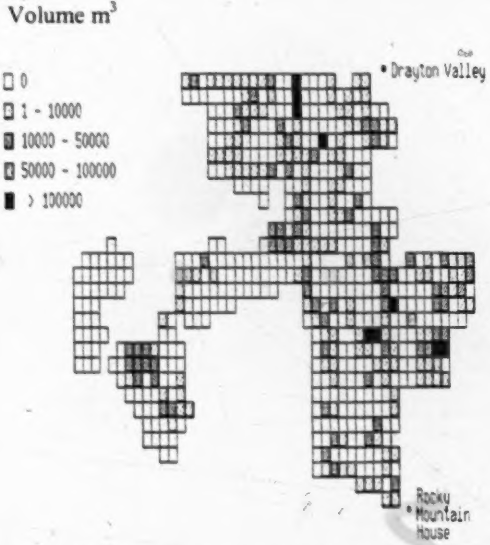
Volume m³
☐ 0
☐ 1 - 10000
▨ 10000 - 50000
☐ 50000 - 100000
■ > 100000
• Drayton Valley
• Rocky Mountain House

Figure 4.5b. Wood procurement zone for Scenario 2 in Period 10. (Number of locations harvested = 229).

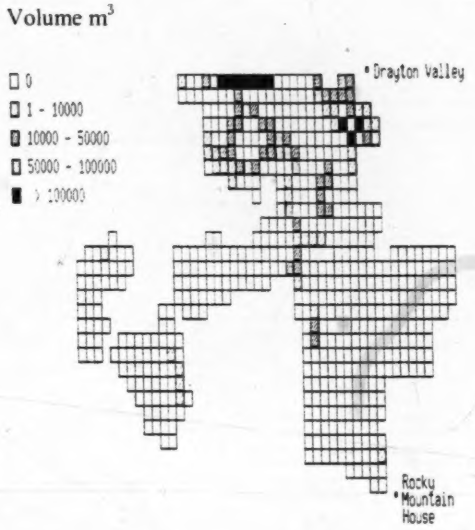
Volume m³
0
1 - 10000
10000 - 50000
50000 - 100000
> 100000
Drayton Valley
Rocky Mountain House

Figure 4.6a. Wood procurement zone for Scenario 3 in Period 1. (Number of locations harvested = 93).

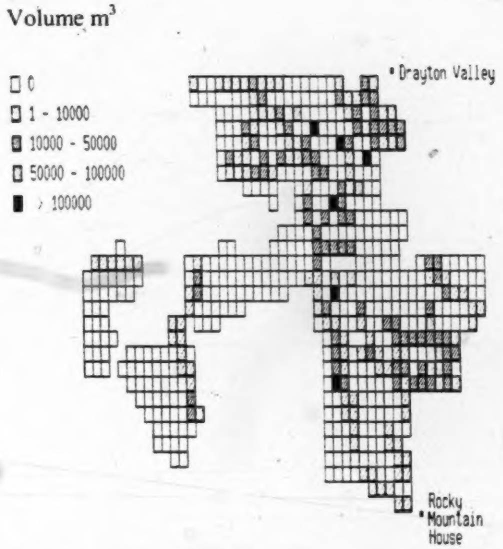
Volume m³
0
1 - 10000
10000 - 50000
50000 - 100000
> 100000
Drayton Valley
Rocky Mountain House

Figure 4.6b. Wood procurement zone for Scenario 3 in Period 10. (Number of locations harvested = 202).

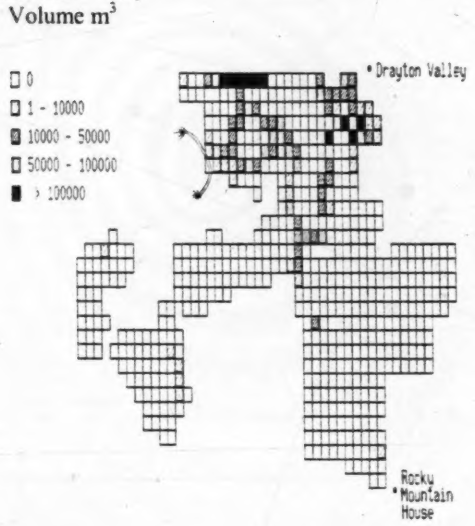
Volume m³
0
1 - 10000
10000 - 50000
50000 - 100000
> 100000
Drayton Valley
Rocky Mountain House

Figure 4.7a. Wood procurement zone for Scenario 4 in Period 1. (Number of locations harvested = 92).

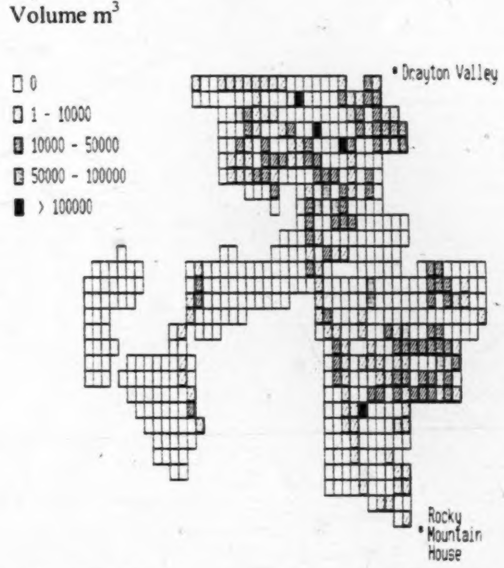
Volume m³
0
1 - 10000
10000 - 50000
50000 - 100000
> 100000
Drayton Valley
Rocky Mountain House

Figure 4.7b. Wood procurement zone for Scenario 4 in Period 10. (Number of locations harvested = 195).

Table 4.5. Total number of locations harvested and the average area (ha) harvested per location by period.

Model run	Planning period										Mean
	1	2	3	4	5	6	7	8	9	10	
Baserun 1											
No. of locations harvested	144	172	188	183	183	184	200	248	234	238	**197**
Average area/location	218	135	120	122	124	138	137	140	183	201	**152**
Scenario 1											
No. of locations harvested	144	170	183	180	180	183	198	232	228	233	**193**
Average area/location	217	136	123	129	130	140	147	158	218	205	**160**
Scenario 2											
No. of locations harvested	144	161	183	176	180	162	196	229	228	229	**188**
Average area/location	217	145	123	130	131	152	148	162	239	218	**167**
Scenario 3											
No. of locations harvested	93	128	145	171	174	157	149	184	213	202	**162**
Average area/location	242	141	120	117	110	133	152	164	189	200	**153**
Scenario 4											
No. of locations harvested	92	123	138	168	173	154	147	183	200	195	**161**
Average area/location	248	142	128	116	116	134	159	172	233	204	**157**

Effect of Timber Harvesting on Hunter Behavior

In the previous section we discussed the influence of hunter preferences for hunting sites on the timber harvest schedule. The question we need to consider next is whether timber harvesting affects hunter behavior (choice of hunting sites) on the landscape and how. We assumed that Drayton Valley and Rocky Mountain House (RMH) were the two points of entry into the FMA not only for hunters originating from these two towns, but also for hunters from surrounding communities. We estimated the probability of hunters visiting any of the 577 locations (using Equation 4.6), and using the average number of hunters, predicted the distribution of hunters on the landscape in each planning period. Since the probability of hunters visiting any site cannot be zero, a decision was made to consider any site with a probability less than 0.001 of being visited as equal to zero. With a 1000 hunters, this probability translates to less than one person visiting the site. This assumption is used in both Tables 4.6 and 4.7. In this model, three specific factors were found to be significant determinants of hunter behavior. These were: the percentage of the area harvested in the last ten years in each location, access, and the habitat suitability index (HSI). The impact of harvesting on hunter behavior was therefore evaluated by examining the relationship between the predicted number of hunters to a site and these characteristics, and the distribution of the hunters on the landscape. Table 4.6 shows a summary of the characteristics of the sites visited by hunters. The most important impact of harvesting on non-timber values is access development. The sites that were visited the most by hunters were those without access. For example, in Scenario 1, only 6.7% of all sites with more than 30 hunters had any access roads. Therefore, access development to facilitate timber harvesting is a major determinant of where hunters will visit. A second factor was the percent area harvested in the last ten years. The higher this percentage, the lower the number of hunters visiting the site. This is because large areas harvested reduce the suitability of the site as a habitat for elk and hence the number of hunters visiting such sites. Another factor in the HSI, which is determined by the percentage canopy cover and the percentage of deciduous cover. Harvesting tends to reduce the HSI for elk and so leads to a reduction in the probability that hunters will visit harvested sites.

Table 4.6. Summary of characteristics of sites chosen by hunters in the first planning period.

	Scenario 1			Scenario 2		
Number of Hunters per location	Average % of locations accessed	Average % of area harvested	Average HSI	Average % of locations accessed	Average % of area harvested	Average HSI
> 30	6.7	0.6	0.2	4.9	0.6	0.2
1 - 30	28.4	4.2	0.1	28.8	4.4	0.1

Figures 4.8 - 4.11 show the distribution of elk hunters on the landscape in the first and last planning periods. The most obvious pattern in all the figures is that most of the locations visited by more than 30 hunters are close to Drayton Valley and Rocky Mountain House. This is expected because the further away a site is from the access towns, the lower the utility for hunters and hence the lower the probability that such a site would be visited.

Table 4.7 shows the distribution of the number of locations visited by hunters in each of the ten planning periods. The table and Figures 4.8 to 4.11 reveal several important patterns regarding the distribution of hunters in space and time on the landscape:

1) Timber harvesting tends to concentrate hunters to fewer, unaccessed locations. This means that when there is less timber harvesting, the hunters are more spread out and when there is more timber harvesting they tend to concentrate in the unaccessed areas. The number of locations visited when there is no harvesting (Baserun 2) is significantly and consistently higher than in scenarios 1, 2 and 4. The number of locations visited by hunters in Baserun 2 remains the same throughout the planning horizon because there is no harvesting in that run, and consequently, the characteristics of the forest remained unchanged. The concentration of hunters to fewer locations due to harvesting increases the number of locations visited by more than 30 hunters. The increased concentration of hunters will lead to congestion of hunters in few locations.

2) The total number of locations visited by hunters decrease over time, that is, from period 1 to period 10 in each model except Baserun 2. This is a response by hunters to the increasing number of locations accessed and harvested over time (Table 4.5). On the other hand, the number of locations visited by more than 30 hunters increase over time except in the last two planning periods. This is a direct consequence of the concentration of hunters to fewer locations.

3) Increasing the weight (δ) on the non-timber benefits from 1 to 5 in the objective function reduces further the total number of locations visited by at least one hunter and increases the number of locations visited by more than 30 hunters. But the fact that the differences between the distribution of hunters in Scenarios 1 and 2 are small may suggest that the distribution of hunters on the landscape is determined more by the characteristics of the sites than the weight on non-timber

values. This is because the weight only increases the magnitude of non-timber values (Table 4.4) on a site without affecting the characteristics of the sites.

4) The total number of locations visited by at least one hunter is inversely related to the level of harvesting on the landscape. This is obvious from Table 4.7, where the harvest level in Scenario 4 is higher than that in Baserun 2 and lower than Scenario 1. The total number of locations visited by at least one hunter in Scenario 4 is greater than that in Scenario 1 but less than Baserun 2. This is also the case with the number of locations visited by more than 30 hunters.

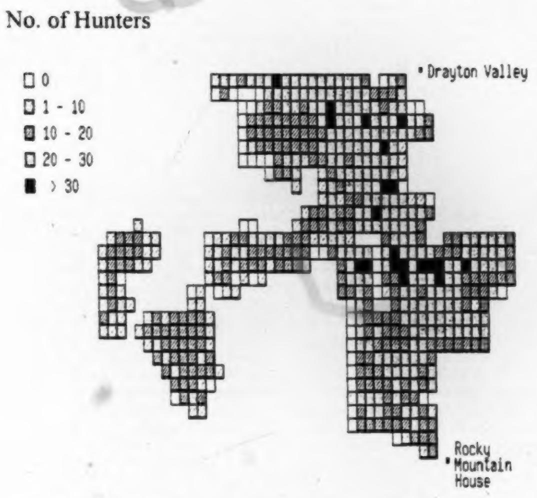
No. of Hunters
0
1 - 10
10 - 20
20 - 30
> 30
Drayton Valley
Rocky Mountain House

Figure 4.8a. Distribution of hunters on the landscape for Baserun 2 in Period 1.

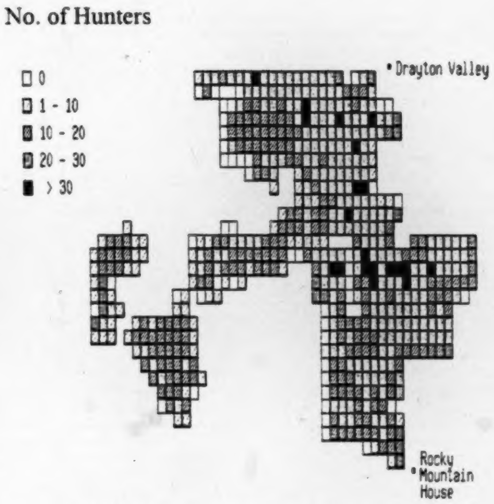
No. of Hunters
0
1 - 10
10 - 20
20 - 30
> 30
Drayton Valley
Rocky Mountain House

Figure 4.8b. Distribution of hunters on the landscape for Baserun 2 in Period 10.

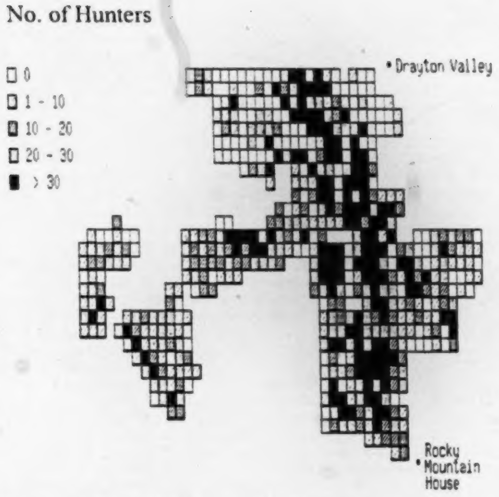
No. of Hunters
□ 0
□ 1 - 10
▨ 10 - 20
□ 20 - 30
■ > 30
Drayton Valley
Rocky Mountain House

Figure 4.9a. Distribution of hunters on the landscape for Scenario 1 in Period 1.

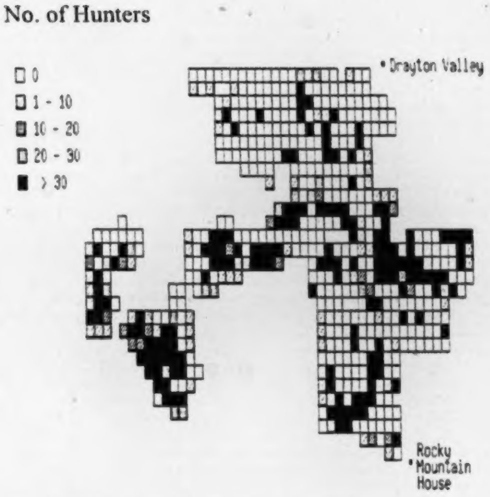
No. of Hunters
□ 0
□ 1 - 10
▨ 10 - 20
□ 20 - 30
■ > 30
Drayton Valley
Rocky Mountain House

Figure 4.9b. Distribution of hunters on the landscape for Scenario 1 in Period 10.

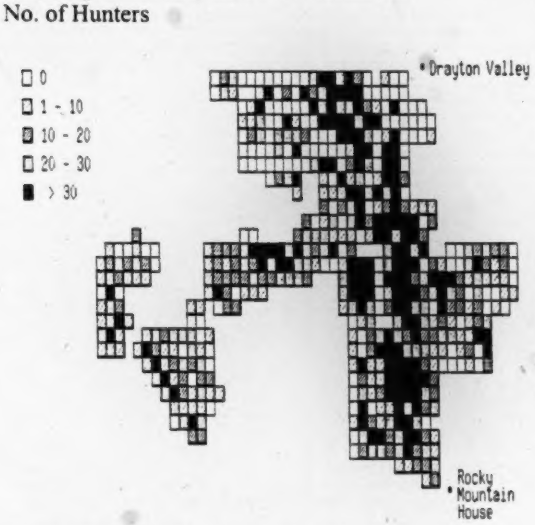
No. of Hunters
□ 0
□ 1 - 10
▨ 10 - 20
□ 20 - 30
■ > 30
Drayton Valley
Rocky Mountain House

Figure 4.10a. Distribution of hunters on the landscape for Scenario 2 in Period 1.

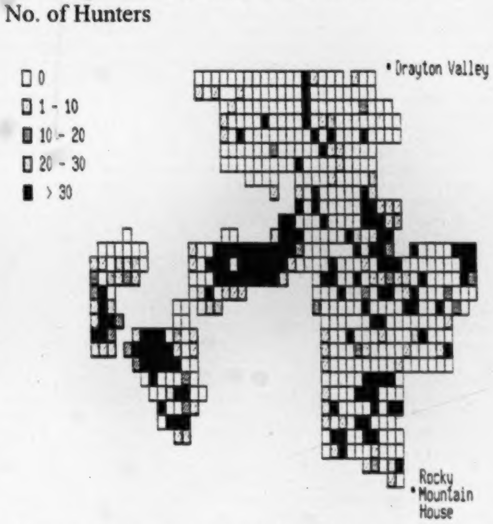
No. of Hunters
□ 0
□ 1 - 10
▨ 10 - 20
□ 20 - 30
■ > 30
Drayton Valley
Rocky Mountain House

Figure 4.10b. Distribution of hunters on the landscape for Scenario 2 in Period 10.

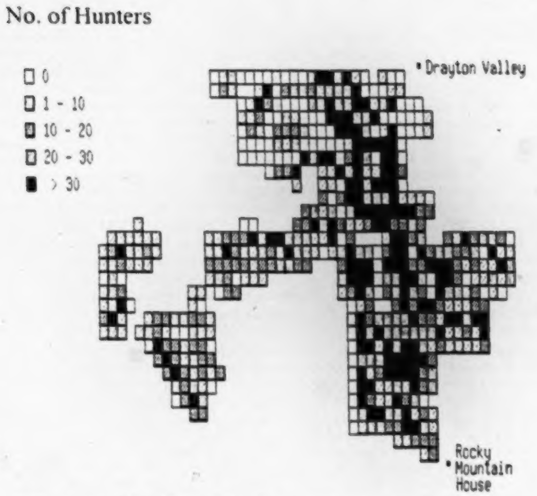

Figure 4.11a. Distribution of hunters on the landscape for Scenario 4 in Period 1.

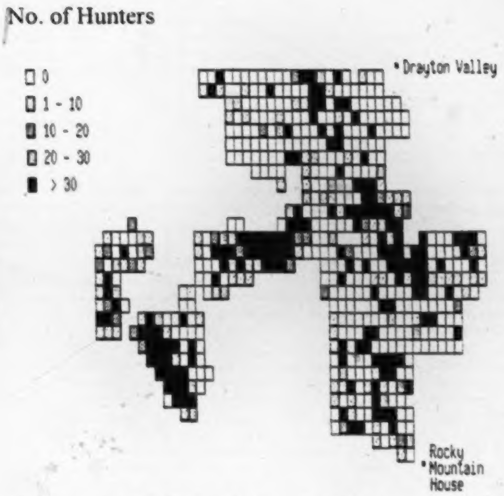

Figure 4.11b. Distribution of hunters on the landscape for Scenario 4 in Period 10.

Table 4.7. The distribution of the number of locations visited by hunters in each period by model run.

Model run	Planning period										Mean
	1	2	3	4	5	6	7	8	9	10	
Baserun 2											
Total visited*	530	530	530	530	530	530	530	530	530	530	**530**
> 30 hunters	20	20	20	20	20	20	20	20	20	20	**20**
Scenario 1											
Total visited	441	385	384	384	386	374	369	303	248	246	**352**
> 30 hunters	135	149	165	157	157	157	163	157	143	151	**153**
Scenario 2											
Total visited	438	382	381	380	380	368	361	302	242	240	**347**
> 30 hunters	138	148	166	168	163	157	167	158	142	133	**154**
Scenario 4											
Total visited	472	424	415	407	404	399	393	342	273	286	**382**
> 30 hunters	132	146	149	153	154	155	154	155	152	153	**150**

*Note: Since the probability of hunters visiting any site cannot be zero, any site with a probability of less than 0.001 of being visited was set to zero.

DISCUSSION

The modeling approach and results presented in this paper have several management applications. An important application of the results presented here is that the model can be used to understand and analyze the tradeoffs inherent in the joint production of timber and non-timber values on the same landscape. While the impacts of non-timber values on the harvest schedule were small because the non-timber values are small relative to timber there are important effects at the margin. Given that the major part of the forest value come from timber harvesting, it seems unlikely that the increased non-timber values due to reduced harvesting can compensate for the loss of timber revenue. However, it should be noted that the recreational hunting in the FMA used here is only a part of the non-timber benefits that the forest can produce. And so it may be possible to compensate for these losses in timber revenue if all non-timber values of the forest can be quantified and included. Restricting timber harvesting to the most valuable stands and concentrating forest management activities in these areas may serve to reduce the margin of tradeoff between timber and non-timber values. This means that timber growing on unaccessed low productivity sites will have more value if left for non-timber purposes than harvested uneconomically. The benefits of doing this is an improvement in timber values by reducing the road building costs, and increase non-timber values by reducing the negative impacts of access roads on non-timber values.

The models provide shadow price information on the demand constraints for lumber and OSB. These constraints represent the marginal costs (or marginal values) of production of these two timber outputs. All marginal costs for lumber increase with time, indicating scarcity of conifer as wood is harvested off the land base. The marginal costs for OSB on the other hand increase only slightly in all of the scenarios examined. This suggests that the current specification of the demand for aspen wood is low, and that the land base is probably capable of sustaining higher harvest levels. In general, inclusion of non-timber values increased marginal costs of both lumber and OSB. The increased marginal costs for models with non-timber benefits seem to be related to the tradeoffs that have to be made to produce non-timber benefits. The significance of these shadow prices is that they provide a way of tying the wood production to marginal costs and values of timber and non-timber, which can be compared to expectations of future timber prices.

Whilst most of the harvesting was limited to locations with permanent access, most hunters on the other hand avoided locations with harvesting activities. This shows an obvious tradeoff between harvesting and hunter preferences for areas with permanent access. This model could therefore form a basis for zoning the FMA into areas of timber production and those areas best suited for non-timber uses. These results suggest that hunters will prefer zones that are close to their towns of origin, have little or no access roads, and minimal harvesting activities. In this model, both access development costs and non-timber benefits favor concentrating harvests. However, concentrating harvests increased the areas harvested per location in order to provide enough wood to meet mill demands. Whilst hunters prefer harvest concentration over the landscape, they avoid large harvested areas within a location. A potential problem therefore is how to balance the conflict between concentrating harvests over the landscape on the one hand, and the reduced sizes of areas harvested in each location on the other, without reducing the annual allowable cut. It is worthy to caution that these results are from a particular application, and so the results may not be universally applicable. The optimal harvest and access schedules that result from incorporating non-timber values into the timber management plan will undoubtedly depend on how the variables incorporated in the random utility model affect predicted hunter utility. How variables are included and which variables are left out may significantly affect the signs of the coefficients on variables in these models. For example, coefficients on road access may be significantly correlated with congestion. Hence, it is important to determine how sensitive harvest and access schedules are to differences in the variables included in RUM models. In addition, if integrated models such as the one presented in this paper are to be used in practice, detailed studies of the target FMAs are required to find the appropriate site choice model.

An important policy implication of these findings is that there appears to be timber management strategies that minimize the impact of timber activities on hunters. Similarly, there may equally be opportunities for influencing hunter behavior that can accommodate a wider scope of timber management. Timber harvesting tends to concentrate hunters to fewer, unaccessed locations. Also, the total number of locations visited by hunters decrease over time in response to the spreading out of timber harvests into previously unaccessed locations. These results suggest that at current levels of timber harvest and hunter use, in the long-term, most suitable sites for hunting could be eliminated, and hunters could be squeezed out of the FMA due to harvesting. But we also know that the most important forestry activities that affect hunter behavior are access construction, amount of area harvested in each accessed location, and the habitat suitability index. By changing these variables, the characteristics of the forest change and so does the behavior of hunters regarding which sites they visit. It is therefore possible that by manipulating road access and harvesting patterns forest managers can influence the distribution of hunters over the landscape. This is very useful in forest planning that seeks to jointly produce timber and non-timber benefits. For example, if the idea is to zone the forest, it should be possible to create conditions that increase hunter utility in zones specifically designed for hunters. On the other hand, it is possible by analyzing various harvest levels and number of hunters expected to visit the FMA, to determine where these hunters might be going and then reduce harvest activities in such areas.

This model can be extended to include hunters from other towns and cities in Alberta. To be able to accurately estimate the value of hunting will require detailed information on the number of hunters visiting the sites within the FMA, their towns of origin, and other socio-economic data. Although our model has been successful at incorporating non-timber values using the RUM, there are still some barriers that have to be cleared before more accurate estimates of non-timber values using this technique can be achieved. First, most site choice models operate on very large scales that do not correspond to the spatial scales of forest management. Therefore, for the models to adequately predict the impacts of forestry activities on hunting behavior, we need to develop recreation models that operate at finer scales. Although not implemented, it would have been possible in our model to use a landscape unit for hunters that is different from the unit for access. For example, we could have used a township as the unit of analysis for hunters and still use 1/9th of a township as an access unit. If this were done, it would allow the use of percentage access as a variable in the RUM model. The current model is limited by the fact that access is either provided to a location or not.

Although a random utility model was used, there is no reason why other forms of non-timber benefits cannot be included in a similar fashion. Also, it is possible to include constraints on the non-timber values and estimate shadow prices for these constraints. Another shortcoming of our model is the lack of feedback with regards to elk population in the system. We expect that as hunters choose the best sites, they reduce the elk populations on such sites, and this should affect the choice of sites with reduced populations by hunters. The present model only handles this situation indirectly, by evaluating elk populations through the habitat suitability index. Future investigations can focus on this area, although we are fully aware that the success of this depends on finding suitable wildlife models that possess this feedback capability and also able to be used in this framework. We believe that the successful implementation of this model provides a basis for further research and extensions of this approach to examine the complex issues of integrating timber and non-timber values in forest management. It is possible to extend the present model by modeling non-timber benefits in the presence of overlapping tenures. This will provide insights into how sustainable forest management will be practiced on the majority of FMA areas in Alberta. The approach can further be extended to model the impacts of non-timber benefits in a forest management strategy such as TRIAD (Seymour and Hunter, 1992). Other forest management problems such as multiple-pass harvesting and decommissioning of roads deserve further investigation, and we believe that these can be examined with minor modifications to the present model.

CONCLUSIONS

This study demonstrates that the decomposition method for solving large-scale forest management problems introduced by Hoganson and Rose is an effective technique for generating acceptable solutions for forest planning models involving non-timber benefits, in the form of a random utility model. In our example application, we were able to find approximately feasible and optimal solutions with average absolute deviations of wood flow from mill demands within 3% in reasonable computational time (20 min). Furthermore, once the code is complete, it is easy to modify and generate solutions to examine different scenarios. Because the model used here is behavioral, it is possible to use it to analyze the impacts of various levels of timber harvests on non-timber benefits and hunter behavior. Similarly, by using different levels of hunters expected to visit the

FMA, it is possible to analyze the effects of hunters on the timber values and harvest schedules. A major achievement of this paper is that it has improved our understanding of the link between landscape characteristics and changes and behavioural responses by hunters. The model presented in this chapter can be further extended to include a feed back in the model for elk populations, and using finer spatial scales to estimate the non-timber benefits. The approach can be extended to model the impacts of non-timber benefits in a TRIAD strategy, and address other forest management problems such as multiple-pass harvesting and decommissioning of roads with minor modifications of the approach presented here.

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CHAPTER 5
SUMMARY AND CONLUSIONS

Three forest management problems in Alberta were analyzed using an optimization approach. The methods used are extensions of the simulation approach by Hoganson and Rose (1984), which is based on dual decomposition techniques. The main advantage of the approach is that it is capable of incorporating a large amount of spatial and temporal detail and is based on the interpretation of the dual side of a linear or non-linear programming formulation of a Model II forest-management scheduling model (Johnson and Scheurman, 1977). A second advantage is that the procedure focuses on shadow prices on the constraints, and allows for optimal scheduling and balancing of multiple products from pure and mixed forest stands. Furthermore, it is possible to model multiple markets and demands in different locations and has the ability to include non-linearities such as; binary variables for modeling forest access and non-linear product demands. All the models in this thesis were solved on a personal computer with a Pentium III 500 Mhz microprocessor. A general conclusion from all three papers is that the decomposition method is effective for generating optimal near feasible solutions within a short computer runtime of about 20 – 30 min. The method therefore has a potential of being applied in practice to investigate long-term timber supply and demand situations where spatial and temporal detail are required.

The objectives of the first paper were to: i) extend the dual decomposition approach to incorporate the types of constraints implied by overlapping tenure, and ii) estimate the costs associated with various constraints implied by overlapping tenures in Alberta. The Drayton Valley and Edson Forest Management Agreement areas (FMAs) of Weyerhaeuser Canada were used as the study area. A Model II forest-scheduling model that maximized net present value subject to mill capacity, regeneration, area, and overlapping tenure constraints was specified. The resulting formulation was extremely large with over 5 million decision variables and about 120,000 constraints.

Two major conclusions were drawn from the overlapping tenure application of the model in the second Chapter. First, constraints imposed by overlapping tenures were shown to lead to inefficiencies in wood allocation and consequently, substantial increases in the marginal costs of production. Secondly, the effect of the overlapping tenure constraints was unevenly distributed among mills. The results were contrary to a common sense expectation that removal of constraints would lead to reduction in marginal costs for all mills that share harvesting rights on the FMAs. Mills that are located far away (outside of the locations from which they would have harvested wood without any constraints) from their allowed harvest locations, gained when the constraints were removed. On the other hand, mills that have harvesting rights to locations within their economic woodsheds, appeared to incur losses following the removal of constraints. Efficient wood allocation may be achieved by eliminating all harvest area restrictions. While the results here suggest that overlapping tenure constraints should be removed and better ways of allocating land for harvest should be sought, they also suggest that in some cases, removal of overlapping tenure constraints may decrease flexibility for some mills resulting in increases in costs for those mills when overlapping tenure constraints are removed. This suggests that although policies to remove overlapping tenure constraints would be efficient, they will be opposed by some tenure holders that derive an economic advantage from existing arrangements. In these cases some means of compensating mills that lose as a result of more efficient wood allocation may have to be arranged. Because the model described in this paper dealt only with timber supply in the context of overlapping tenure constraints, the model was extended to consider access road costs in Chapter 3, and non-timber values in the Chapter 4.

In chapter 3, we developed a mixed-integer non-linear programming (MINLP) model that integrates access road development with forest harvest scheduling for the Drayton Valley FMA of Weyerhaeuser Canada. The resulting formulation, with about 2.6 million decision variables and over 95,000 constraints, included 3850 integer variables and constraints. The model was solved within 20 minutes on a personal computer, which is a reasonably short computer time, given the size of the model. Two model runs, one including a road building cost of $20,000/location and the other with zero road building costs, were examined.

The inclusion of road access costs in forest management scheduling revealed a concentration of forest management activities to fewer locations over the planning period compared to when road construction costs are zero. Consequently, positive access costs resulted in a decrease in the number and frequency of locations accessed and harvested during the planning horizon. The shadow price information on the mill demand and access constraints over time give insights into future production costs or timber prices, which are valuable for determining supply planning as well as silvicultural and road building investment decisions. For example, the information provided by the model can be used to conduct sensitivity analyses, which will show how much wood can be profitably harvested given a fixed road construction budget, or given a fixed demand, how much it will cost to build roads to satisfy that demand.

Chapter 4 was an extension of Chapter 3 to include a random utility model that measured benefits to elk hunters in the study area. The model incorporated a site choice model into an integrated forest scheduling and access activities model. There were two demand centers: Drayton Valley and Rocky Mountain House. The former town was a demand center for timber and hunters, whilst the latter location was considered a demand center for hunters in communities in and around Rocky Mountain House. Six different scenarios with varying annual allowable cuts for the lumber and OSB mills and different weights on non-timber values were investigated. The solution time was similar to that of Chapter 3, and absolute deviations converged within 20 minutes of runtime.

Comparisons of the timber and non-timber values from Chapter 4 showed that non-timber benefits constituted less than 1% of the net present value of the whole Drayton Valley FMA. The results showed that there are significant tradeoffs between timber and non-timber values. The benefits derived by elk hunters were small compared to timber values, and the increase in recreational values due to reduced harvesting could not compensate for the lost timber revenue. Inclusion of non-timber values only slightly affected the forest management schedules and access road development. On the other hand, timber harvesting significantly influenced hunter behavior. Most hunters avoided sites with access roads and sites with large harvested areas, but preferred sites that were close to their towns of origin, with large areas of forest cover. Timber harvesting tends to concentrate hunters to fewer, unaccessed locations. Furthermore, the total number of locations visited by hunters decrease over time in response to the spreading out of timber harvests into previously unaccessed locations. These results imply that at current levels of timber harvest and hunter use, in the long-term, most suitable sites for hunting could be eliminated, and hunters squeezed out of the FMA due to harvesting. Since the model used in this paper is behavioral, it is possible to use it to analyze the impacts of various levels of timber harvests on non-timber benefits and hunter behavior. Similarly, by using different levels of hunters expected to visit the FMA, it is possible to analyze the effects of hunters on the timber values and harvest schedules. There appear to be flexible timber management techniques that can minimize the impact of timber activities on hunter welfare. But the question is whether there are equally ways of managing hunting opportunities that can accommodate a wider scope of timber management. It is important to point out that the type of schedule that results from incorporating non-timber values into the timber management plan will depend on the how the RUM is estimated, and how the variables in the RUM affect hunter utility. It is therefore important that for purposes of implementing actual management plans, detailed studies of the target FMA needs to be done to estimate an appropriate RUM.

Despite the successful implementation of these three models, and the insights the results have provided, there are still several extensions and unanswered questions that need to be investigated. It is possible to extend the model based on the framework presented here to include a feedback in the model for elk populations, and using finer spatial scales to estimate the random utility model. The approach can also be extended to model the impacts of non-timber benefits in a TRIAD landscape management strategy, and address other forest management problems such as multiple-pass harvesting and decommissioning of roads. These will require further modifications of the approach presented here.

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APPENDICES

APPENDIX I

CALCULATION OF THE NUMBER OF CONSTRAINTS AND DECISION VARIABLES FOR THE ACCESS PROBLEM

Table A1. Calculation of the number of constraints for the problems in Chapters 3 and 4.

Eqn #	Constraint type	Constraint calculation		Number of constraints
1	Sawmill demand	1 sawmill	x10 periods	10
2	OSB mill demand	2 OSB mills	x10 periods	20
3	Equation [3.2]	577 locations	x10 periods	5,770
4	Equation [3.3]	385 locations	x10 periods	3,850
5	Equation [3.4]	192 locations	x10 periods	1,920
6	Access constraint (Equation [3.5])	385 locations	x10 periods	3,850
7	Initial area constraints	18833 analysis areas		18,833
8	Area harvested = area regenerated	6156 stand types	x10 periods	61,560
Total				95,613

Table A2. Calculation of the number of decision variables for the problems in Chapters 3 and 4.

Variable Types	Birth period	Number of periods	Number of shipping alternatives	Number of prescriptions	Number of stand types or analysis areas	Number of decision variables
Initial Stands						
Harvesting variables		6	6	1	18,833	677,988
Ending inventory		1		1	18,833	18,833
Integer Variables		10				3,870
Shipping alternatives for non-POLS		10	8		385 locations	30800
Regeneration stands						
Harvest and regeneration variables	1	(10-4-1)	6	3	6156 stand types	554,040
	2	(10-4-2)	6	3	6156 stand types	443,232
	3	(10-4-3)	6	3	6156 stand types	332,424
	4	(10-4-4)	6	3	6156 stand types	221,616
	5	(10-4-5)	6	3	6156 stand types	110,808
	6	0				
	7	0				
	8	0				
	9	0				
	10	0				
Ending inventory		10		3	6156 stand types	184,680
Total						2,578,291

APPENDIX II

LIST OF RELEVANT DATA USED IN THE THESIS

Table A3. Harvests costs by Period (each period corresponds to 10 yrs).

Age (periods)	4	5	6	7	8	9	10	11	12
Harvest cost ($/m^3)	19	18	17	16	15	14	14	13	13
Age (periods)	13	14	15	16	17	18	19	20	
Harvest cost ($/m^3)	13	12	12	12	12	11	11	10	

Table A4. Demand locations, product types, and product recovery factors from roundwood.

Mill Number	Mill Location	Mill Ownership	Mill type /species type	Size	End Product	Recovery factors*	Processing cost $/ m^3
1	Drayton	Weyerhaeuser	Sawmill	Sawtimber	Lumber	0.55	30
	Valley		Pine/spruce		Chips	0.36	15
				Merch.Vol	Lumber	0.40	35
					Chips	0.30	20
2	Edson	Edson	Sawmill	Sawtimber	Lumber	0.55	30
		Timber	Pine/spruce		Chips	0.36	15
				Merch.Vol	Lumber	0.40	35
					Chips	0.30	20
3	Whitecourt	Millar	Sawmill	Sawtimber	Lumber	0.55	30
		Western	Pine/spruce		Chips	0.36	15
				Merch.Vol	Lumber	0.40	35
					Chips	0.30	20
4	Drayton	Weyerhaeuser	OSB mill	Sawtimber	OSB	0.85	20
	Valley		Aspen	Merch.Vol	OSB	0.80	30
5	Edson	Weyerhaeuser	OSB mill	Sawtimber	OSB	0.85	20
			Aspen	Merch.Vol	OSB	0.80	30
6	Whitecourt	Millar	Pulp mill	Sawtimber	Chips	0.95	15
		Western	Pine/spruce	Merch. vol	Chips	0.90	20

*Note: Recovery factors are defined as m^3 of end product per m^3 of roundwood input.

Table A5. Age Class Distribution (ha) by Forest type for the Edson Forest Management Agreement Area.

Forest type	Areas in hectares by 20-Year Age classes										
	1-20	21-40	41-60	61-80	81-100	101-120	121-140	141-160	161-180	181-200	Total
CPG	0	28	660	2927	471	4299	1698	53	0	0	10137
CPM	0	0	541	4880	1166	23014	1409	493	10	0	31512
CPF	0	3	132	237	15	1328	138	23	41	0	1918
CPU	0	0	0	0	59	31	0	0	0	0	90
CSwG	0	59	465	1783	3232	1577	1981	105	0	0	9201
CSwM	0	1	703	759	1025	5637	3865	1625	46	0	13661
CSwF	0	25	36	18	11	483	49	84	4	0	709
CSwU	0	0	2	0	0	2	0	0	0	0	3
CSbG	0	0	430	689	1982	5512	2656	664	25	0	11958
CSbM	0	0	1256	2471	3066	13108	5657	4756	238	146	30699
CSbF	0	0	79	2793	1167	4232	2199	1714	100	36	12319
CSbU	0	2	1604	13434	5498	17924	4946	6015	906	579	50908
CDPG	0	27	85	770	43	873	452	73	0	0	2323
CDPM	0	2	74	1644	180	1587	307	162	0	0	3956
CDPF	0	0	4	78	0	0	0	0	0	0	81
CDPU	0	0	0	0	0	0	0	0	0	0	0
CDSwG	0	35	343	1438	1979	1159	293	0	0	0	5247
CDSwM	0	0	311	489	434	3887	1899	1412	13	0	8445
CDSwF	0	0	5	2	0	60	7	63	6	0	143
CDSwU	0	0	2	0	0	0	0	0	0	0	2
CDSbG	0	0	2	117	157	268	36	53	0	0	634
CDSbM	0	0	2	80	58	149	59	76	20	10	455
CDSbF	0	0	0	21	10	44	0	0	0	0	75
CDSbU	0	0	0	7	9	11	6	4	0	0	37
DAwG	1239	4379	2397	11450	10125	6257	936	28	0	0	36812
DAwM	2	2183	2326	15303	5677	21157	3166	1069	0	0	50885
DAwF	0	99	301	117	19	1129	9	830	16	0	2521
DAwU	0	0	9	4	0	4	0	2	0	0	20
DCAwG	19	783	616	2255	2097	1821	165	0	0	0	7757
DCAM	0	593	1098	4045	1573	9232	757	525	0	0	17824
DCAwF	0	13	218	80	121	667	64	1122	0	0	2285
DCAwU	0	0	15	5	2	45	0	35	0	0	103
Total	1260	8231	13718	67900	40178	125479	32756	20987	1423	771	**312722**

Table A6. Age Class Distribution (ha) by Forest type for the Drayton Valley Forest Management Agreement Area.

Forest type	Areas in hectares by 20-Year Age classes									
	1-20	21-40	41-60	61-80	81-100	101-120	121-140	141-160	161-180	181-200	Total
CPG	0	0	0	28	35	152	0	0	0	0	215
CPM	0	0	0	8	30	147	0	0	0	0	184
CPF	0	0	0	29	10	0	0	0	0	0	39
CPU	0	0	0	0	0	0	0	0	0	0	0
CSwG	0	74	132	1335	703	1477	1047	138	0	0	4906
CSwM	22	6	369	1692	2843	4731	5217	1761	194	24	16859
CSwF	0	5	196	1039	802	1653	546	4679	3036	880	12836
CSwU	1	0	33	10	58	281	20	608	322	56	1390
CSbG	0	25	78	216	237	1487	465	23	0	0	2532
CSbM	139	160	228	1126	887	7040	4330	2337	234	11	16493
CSbF	16	12	2	1210	1129	3015	2610	1107	308	90	9500
CSbU	10	97	1276	8698	4674	8345	5529	3650	933	48	33261
CDPG	0	0	10	9	60	225	0	0	0	0	305
CDPM	0	0	0	0	0	75	0	0	0	0	75
CDPF	0	0	0	0	0	0	0	0	0	0	0
CDPU	0	0	0	0	0	0	0	0	0	0	0
CDSwG	0	49	116	731	645	1084	685	0	0	0	3309
CDSwM	0	11	235	813	1296	2845	2907	468	26	0	8601
CDSwF	0	0	18	385	153	64	9	33	37	0	698
CDSwU	0	0	0	0	0	0	0	0	0	0	0
CDSbG	0	3	26	13	12	75	4	0	0	0	133
CDSbM	0	0	34	33	67	61	55	9	0	0	260
CDSbF	0	0	0	53	10	22	8	0	0	0	92
CDSbU	0	0	13	92	27	38	19	5	0	3	198
DAwG	223	2261	3784	7929	13184	16222	174	0	0	0	43777
DAwM	830	1961	6756	7297	5453	9972	2425	140	0	0	34835
DAwF	54	49	275	248	66	288	240	15	0	0	1236
DAwU	6	41	0	17	1	155	0	0	0	0	220
DCAwG	74	797	901	2045	4280	6066	109	0	0	0	14272
DCAM	247	891	2458	1964	1877	7179	1881	136	0	0	16632
DCAwF	1	2	93	40	38	449	396	22	7	0	1047
DCAwU	0	7	3	13	0	26	0	0	0	0	49
	1625	6450	17035	37071	38577	73176	28677	15133	5098	1113	**223955**

Table A7. The Equations and coefficients used to estimate the Yield for the various species.

Volume	Eco-region	Cover group	Density	Function Form	B0	B1	B2	B3	Z1
Total Volume	All	C, CD	A/B	TGMVOL=(B0+b1*z1+b2*dep_si)*dep_AGE*EXP(-0.006065*dep_AGE)+b3	4.17758	1.92345	0.22222	-307.86918	0
			A/B		4.17758	1.9234	0.22222	-307.86918	1
	All	C, CD	C/D	TGMVOL=(B0+b1*z1+b2*dep_si)*dep_AGE*EXP(-0.006065*dep_AGE)+b3	4.76178	0.285298	0.34274	-290.74806	0
			C/D		4.76178	0.285298	0.34274	-290.74806	1
	All	D,DC	A/B	TGMVOL=(B0+b1*z1+b2*dep_si)*dep_AGE*EXP(-0.006300*dep_AGE)+b3	2.49519	1.977949	0.19506	-163.00859	0
			A/B		2.49519	1.977949	0.19506	-163.00859	1
	All	D,DC	C/D	TGMVOL=(B0+b1*z1+b2*dep_si)*dep_AGE*EXP(-0.006300*dep_AGE)+b3	4.52000	0.460458	0.37321	-293.13037	0
			C/D		4.52000	0.460458	0.37321	-293.13037	1
Conifer Volume	LF	C, CD	A/B	CGMVOL=(b0+b1*DEP_pcon)*tgmvol*(1-exp(-b2*DEP_age))	0.34076	0.075333	0.01171		
		C, CD	C/D	CGMVOL=(b0+b1*DEP_pcon)*tgmvol*(1-exp(-b2*DEP_age))	0.1341	0.0706	10.3023		
	UF	C, CD	A/B	CGMVOL=(b0+b1*DEP_pcon)*tgmvol*(1-exp(-b2*DEP_age))	0.88419	0.00984	954.716		
		C, CD	C/D	CGMVOL=(b0+b1*DEP_pcon)*tgmvol*(1-exp(-b2*DEP_age))	0.84109	0.016692	0.0340		
Decid. Volume	All	D, DC	A/B	DGMVOL=(b0+b1*DEP_pcon)*tgmvol*(1-exp(-b2*DEP_age))	1.03181	-.091384	0.020998020		
		D, DC	C/D	DGMVOL=(b0+b1*DEP_pcon)*tgmvol*(1-exp(-b2*DEP_age))	0.87616	-.0690972	0.053847155		

The variables in the Table are defined as:

Tgmvol: total gross merch volume

Cgmvol: conifer gross merch volume

Dgmvol: deciduous gross merch volume.

Dep_age: simply age

Dep_pcon: percent conifer (0 to 10)

Covergroup

C: Percent conifer = 10, 9, 8

CD: Percent conifer = 7, 6, 5

DC: percent conifer = 4, 3

D: percent conifer = 2, 1, 0

APPENDIX III

Algorithm for Implementing the Overlapping Tenures Model in Chapter 2

The solution of the model as described by Hoganson and Rose (1984) proceeds with the following steps:

1) Use prior information about the problem to estimate the marginal costs of production for each output [lumber, OSB, and chips] and period (that is, the m_{mt}'s).
2) Provide initial estimates of the shadow prices on the harvest area and land base restrictions [the λ_{mt}] (or just set them to zero).
3) Obtain the net marginal costs of final products as: $m_{mt} - \lambda_{mt}$
4) Assume the net m_{mt} estimates are correct and solve Equation 2.8 for the remaining dual variables (a_{is}'s and a^{r}_{is}'s). This basically finds the maximum land value for each stand type.
5) Determine the primal solution (the x_{ijst}'s) in Equation (2.1) that corresponds to the optimal dual solution. This primal solution is not necessarily a feasible solution.
6) Determine the output levels for the primal solution found in Step 5 so that the primal solution can be tested for feasibility.
7) Test for primal feasibility. If the output levels determined in Step 6 are close to their desired output levels (M_{mt}'s), stop, the primal solution is both an optimal and a near-feasible solution.

Otherwise:

8) Use the output levels determined in step 6 and a basic understanding of the relationship between output levels and marginal revenue of production to re-estimate the m_{mt} values. The shadow prices on the demand constraint were

adjusted based on the procedure presented by Hoganson and Rose (1984) and modified by Hauer (1993). This price adjustment is based on a function similar to that shown in Figure A1. The function *f* is piecewise linear which gives price adjustments as a function of the deviation. The function gives large price changes for large deviations and small price changes for small deviations.

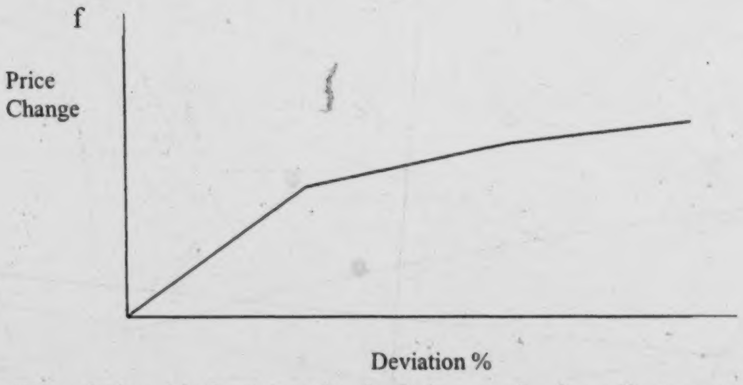

Figure A1. The relationship between the deviations and price adjustments for the demand constraints.

9) Adjustment procedures for the shadow prices on the harvest area constraints (λ_{mt}) is as follows:

   Let F^R_{mt} denote the flow to mill *m* in period *t*, then,

$$F^R_{mt} = -\sum_{l\in M^R}\sum_{i\in I^m}\sum_{s} v_{ijst}x_{ijst} \geq 0$$

   The negative sign indicates that this flow cannot be positive, therefore F^R_{mt} is either zero or negative. Adjustments to the shadow price on this constraint is a function of this flow, denoted by $f(F^R_{mt})$, which is piecewise linear. Large negative flows correspond to large adjustments to the shadow price and vice versa. The adjustment can be written as: $\lambda^1_{mt} = \lambda^0_{mt} + f(F^R_{mt})$

   The nature of this price adjustment curve is shown in Figure A2.

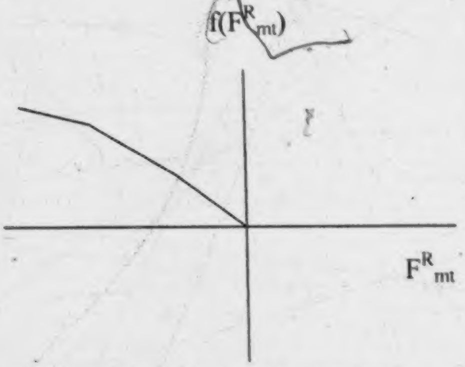

Figure A2. The relationship between the deviations and price adjustments for the overlapping tenure constraints.

10) Return to Step 4.

APPENDIX IV

Algorithm for Implementing the Access Model in Chapter 3

1. Assume that $z_{jt} = 0$, for all $J \notin J^P$. That is, all non-permanently accessed locations are initially closed.
2. Provide initial guesses of timber prices for all t (π_{it}'s)
3. For all permanently assessed locations, ($J \in J^P$), we calculate the value of wood at each mill. That is: $\theta_{jt} = \max_i(\pi_{it} - c^s_{kjt})$. This is done for each of the six wood products.
4. For all non-permanently assessed locations, $J \notin J^P$, we defined sub-destinations on the way to the mills at locations that are permanently accessed and adjacent to areas that are not accessed. The price of wood in each subdestination is calculated by solving iteratively the dynamic programming formulation given by

   $$u_{jt} = \max_{k \in I^B_j}\left\{u_{kt} - c^s_{jkt}\right\} + \lambda_{jt}(z_{jt} - 1) \qquad [A1]$$

   This Equation says that the marginal value of wood at location j in period t is equal to the maximum of the net value of shipping the wood to all adjacent locations minus the shadow cost of opening the location j if location j is not opened. The equation is solved as follows:

   a) List the supply locations that do not have permanent access according to how far they are from permanent access. Locations directly adjacent to permanent access are listed first- followed by areas that are one location away from permanent access, followed by areas 2 locations away, etc.

   b) Initialize u_{jt}'s for each location to zero.

   c) For each time period and location: Solve A1 by setting the initial estimates of u_{kt} to zero. Use the latest update for λ_{jt} and z_{jt} . This will give a u_{jt} , and we also store the best destination (i.e. best supply location).

   d) Compare this iteration's u_{jt}'s to previous iterations. If they are the same or didn't change much then go to Step e. Otherwise repeat Step c.

e) Stop.

The u_{jt}'s and v_{jt}'s in this formulation are functions of the prices we calculate at permanently accessed areas. So there is no iteration procedure other than the one described above needed to re-estimate the prices. The re-estimate of u_{jt}'s are based on re-estimates of π_{it}'s.

5. Assume the π_{it}'s are correct and solve Equation 3.14 and 3.15 for the remaining dual variables (a_{sj}'s and s_{sj}'s).
6. Determine the primal solution (the x_{sjt}'s) in Equation 3.1 that corresponds to the optimal dual solution. This primal solution is not necessarily a feasible solution.
7. Determine the output levels for the primal solution found in Step 6 by summing up all flows of wood through a given location in period *t*. Notice that this flow is not only from harvest within the associated supply location but also from other locations that ship wood through those subdestinations.

8. Test for primal feasibility. If the output levels determined in step 6 are close to their desired output levels (y^s_{jt}), stop, the primal solution is both an optimal and a near-feasible solution.

Otherwise:

9. Use the output levels determined in step 7 and a basic understanding of the relationship between output levels and marginal revenue of production to re-estimate the π_{it} values. The π_{it}'s were adjusted using the methods presented by Hoganson and Rose (1984) and modified by Hauer (1993), similar to Figure A1. Also update the shadow prices on the access constraint as follows:
   a) calculate the approximate value of the shadow price on the access constraint using $\lambda_{jt} = c^A_{jt} / y^s_{jt}$

   where λ_{jt} as the average cost of opening up a closed location, and c^A_{jt} is the fixed cost of accessing a closed location.

   b) Calculate a deviation for this constraint as: $dev_{jt} = y^s_{jt} - y^s_{jt} z_{jt}$,

c) if $dev_{jt} = y^{s}_{jt} - y^{s}_{jt} z_{jt} > 0$, then we adjust $\lambda^{1}_{jt} = \lambda^{0}_{jt}, + f^{g}(dev_{jt})$

d) If the $dev_{jt} = y^{s}_{jt} - y^{s}_{jt} z_{jt} = 0,\ y^{s}_{jt} > 0,\quad z_{jt} = 1$, then the price changes are based on $\lambda^{1}_{jt} = \lambda^{0}_{jt} + f^{0}\left(\frac{c^{A}_{jt}}{y^{s}_{jt}} - \lambda^{0}_{jt} \right)$. The nature of these price adjustment curves are shown as Figures 3.1 and 3.2 in Chapter 3 of the main text.

10. Update the z_{jt}'s. if after the price adjustments above $\lambda_{jt} y^{s}_{jt} \geq c^{A}_{jt}$, then update $z_{jt} = 1$ otherwise, $z_{jt} = 0$

11. Return to step 5

APPENDIX V

Algorithm for Implementing the Random Utility Model in Chapter 4

1. Assume that $z_{jt} = 0$, for all $J \notin J^P$. That is, all non-permanently accessed locations are initially closed.
2. Provide initial guesses of timber prices for all t (π_{it}'s)
3. For all permanently assessed locations, ($j \in J^P$), we calculate the value of wood at each supply location j. That is: $\theta_{jt} = \max_i(\pi_{it} - c^s_{jit})$ (note in the algorithm this is done for each of 6 wood products).
4. For all non-permanently assessed locations, $j \notin J^P$, we defined sub-destinations on the way to the mills at locations that are permanently accessed and adjacent to areas that are not accessed. The price of wood in each subdestination is calculated by solving iteratively the dynamic programming formulation given by

$$u_{jt} = \max_{k \in I_j^B}\left\{u_{kt} - c^s_{jkt}\right\} + \lambda_{jt}(z_{jt} - 1) \qquad \text{[A2]}$$

This Equation says that the marginal value of wood at location j in period t is equal to the maximum of the net value of shipping the wood to all adjacent locations minus the shadow cost of opening the location j if location j is not opened. The equation is solved as follows:

   a. List the supply locations that do not have permanent access according to how far they are from permanent access. Locations directly adjacent to permanent access are listed first- followed by areas that are one location away from permanent access, followed by areas 2 locations away, etc.
   b. Initialize u_{jt}'s for each location to zero.
   c. For each time period and location: Solve A2 by setting the initial estimates of u_{kt} to zero. . Use the latest update for λ_{jt} and z_{jt} . This will give a u_{jt} , and we also store the best destination (i.e. best supply location).

d. Compare this iteration's $u_{jt}'s$ to previous iterations. If they are the same then go to Step e. Otherwise repeat Step c.

e. Stop.

The $u_{jt}'s$ and $v_{jt}'s$ in this formulation are functions of the prices we calculate at permanently accessed areas. So there is no iteration procedure other than the one described above needed to re-estimate the prices. The re-estimate of $u_{jt}'s$ are based on re-estimates of π_{it}'s.

5. Assume the $\pi_{it}'s$ are correct and solve Equation 4.18 for the remaining dual variables ($a_{js}'s$ and $s_{js}'s$). Here the solution for the remaining dual variables includes the non-timber values as specified in Equations 4.20 and 4.21 in the text. To obtain the dual variables we solve the following in order: The optimal rotation problems using last period T prices:

5.1

a. Calculate the average $\dfrac{e^{X_{hjt}\cdot\mathrm{B}}}{\sum_{k=1}^{J} e^{X_{hkt}\cdot\mathrm{B}}}$ for each location j over the planning horizon. Call this $\overline{\pi}_j = \dfrac{1}{T}\sum_{t=1}^{T}\dfrac{e^{X_{hjt}\cdot\mathrm{B}}}{\sum_{k=1}^{J} e^{X_{hkt}\cdot\mathrm{B}}}$.

b. Calculate optimal rotation ages for each land type based on:

$$s_j = \max_t \left(\frac{\left(-c^h\beta^t - c^r + u_{jT}v_{jt}\beta^t + \dfrac{\delta\beta^{t-1}}{\beta_0}\sum_{h=1}^{H} n_h \left(\begin{array}{c} \sum_{t'=0}^{t-1}\left(\beta_3 HSI_{jt'}\overline{\pi}_j\right) \\ +\sum_{t'=t}^{t+1}\left(\dfrac{\beta_2}{A_{jt}}\overline{\pi}_j\right) \end{array} \right) \right)}{1-\left(1+r\right)^{-t}} \right)$$

choose a from 1 to the maximum rotation length. Calculate permanent reserve value by setting a = maximum age and dropping the first three terms from the equation above (i. e., drop $-c^h\beta^a - c^r + u_{jT}v_{ja}\beta^a$)

c. Calculate the land value of bareland for each period t beyond the planning horizon: $s_{js} = s_j \times \beta^s$ for s=*T+1,…,MaxPeriods*

d. Solve the bareland harvest scheduling problems.

e. $$s_{js} \geq -c_{sjt} + u_{jt}v_{sjt} + s_{jt} + \frac{\delta\beta^{t-1}}{\beta_0}\sum_{h=1}^{H} n_h \left(\begin{array}{l} \displaystyle\sum_{t'=s}^{t-1}\left(\beta_3 HSI_{jst'} \frac{e^{X_{hjt}\cdot\mathbf{B}}}{\sum_{k=1}^{J} e^{X_{hkt}\cdot\mathbf{B}}} \right) + \\ \displaystyle\sum_{t'=t}^{t+1}\left(\frac{\beta_2}{A_{jt}} \frac{e^{X_{hjt}\cdot\mathbf{B}}}{\sum_{k=1}^{J} e^{X_{hjt}\cdot\mathbf{B}}} \right) \end{array} \right)$$

$$if\ x_{sjt} > 0 \quad \forall\, j, s = 0,\dots,T\ and\ t = s + z,\dots, MaxRotation$$

Note for t>T you replace $\frac{e^{X_{hjt}\cdot\mathbf{B}}}{\sum_{k=1}^{J} e^{X_{hkt}\cdot\mathbf{B}}}$ with $\bar{\pi}_j = \frac{1}{T}\sum_{t=1}^{T}\frac{e^{X_{hjt}\cdot\mathbf{B}}}{\sum_{k=1}^{J} e^{X_{hkt}\cdot\mathbf{B}}}$ in the above equation. Note also calculate a reserve option by setting t = *MaxRotation* and calculating the value of non-timber benefits. *S_{js} is the max of the above timber options and the Reserve value.*

f. Solve the existing stand harvest schedule problems

$$a_{sj} \geq -c_{sjt} + u_{jt}v_{sjt} + s_{jt} + \frac{\delta\beta^{t-1}}{\beta_0}\sum_{h=1}^{H} n_h \left(\sum_{t'=s}^{t-1}\left(\beta_3 HSI_{jst'} \frac{e^{X_{hjt}\cdot\mathbf{B}}}{\sum_{k=1}^{J} e^{X_{hkt}\cdot\mathbf{B}}} \right) + \sum_{t'=t}^{t+1}\left(\frac{\beta_2}{A_{jt}} \frac{e^{X_{hjt}\cdot\mathbf{B}}}{\sum_{k=1}^{J} e^{X_{hjt}\cdot\mathbf{B}}} \right) \right)$$ Note for

$$if\ x_{sjt} > 0 \quad \forall\, j, s = -T,\dots,0\ and\ t = 0,\dots, MaxRot$$

t>T you replace $\frac{e^{X_{hjt}\cdot\mathbf{B}}}{\sum_{k=1}^{J} e^{X_{hkt}\cdot\mathbf{B}}}$ with $\bar{\pi}_j = \frac{1}{T}\sum_{t=1}^{T}\frac{e^{X_{hjt}\cdot\mathbf{B}}}{\sum_{k=1}^{J} e^{X_{hkt}\cdot\mathbf{B}}}$ in the above equation. Note also calculate a reserve option by setting t=*MaxRotation* and calculating the value of non-timber benefits. *a_{js} is the max of the above timber options and the Reserve value.*

6. Determine the primal solution (the x_{sjt}'s) in Equation 4.10 that corresponds to the optimal dual solution. This primal solution is not necessarily a feasible solution.

7. Determine the output levels for the primal solution found in Step 6 by summing up all flows of wood through a given location in period t. Notice that this flow is not only from harvest within the associated supply location but also from other locations that ship wood through those subdestinations.

8. Test for primal feasibility. If the output levels determined in step 6 are close to their desired output levels (y^{s}_{jt}), stop, the primal solution is both an optimal and a near-feasible solution.

Otherwise:

9. Use the output levels determined in step 7 and a basic understanding of the relationship between output levels and marginal revenue of production to re-estimate the π_{it} values. The π_{it} were adjusted using the methods presented by Hoganson and Rose (1984) and modified by Hauer (1993). Also update the shadow prices on the access constraint as follows:

    a. calculate the approximate value of the shadow price on the access constraint using Equation 4.22

    with λ_{jt} as the average cost of opening up a closed location, and c^{A}_{jt} is the fixed cost of accessing a closed location.

    b. Calculate a deviation for this constraint as: $dev_{jt} = y^{s}_{jt} - y^{s}_{jt} z_{jt}$,

    c. if $dev_{jt} = y^{s}_{jt} - y^{s}_{jt} z_{jt} > 0$, then we adjust $\lambda^{1}_{jt} = \lambda^{0}_{jt}, + f(dev_{jt})$

    d. If the $dev_{jt} = y^{s}_{jt} - y^{s}_{jt} z_{jt} = 0,\ y^{s}_{jt} > 0,\ \ z_{jt} = 1$, then the price changes are based on

$$\lambda^{1}_{jt} = \lambda^{0}_{jt} + f\left(\frac{c^{A}_{jt}}{y^{s}_{jt}} - \frac{1}{y^{s}_{jt}} \frac{\delta}{\beta_0} \sum_{h}^{H} n_h \left(\beta_1 \log e^{(1,X)_{hjt}\mathrm{B}} - \beta_1 \log e^{(0,X)_{hjt}\mathrm{B}} \right) - \lambda^{0}_{jt} \right).$$

    Note: this must be modified to account for non-timber benefits.

10. Update the z_{jt}'s. if after the price adjustments above

$$\lambda_{jt} y^{s}_{jt} + \frac{\delta\beta^{t-1}}{\beta_0} \sum_{h}^{H} n_h \left(\beta_1 \log e^{(1,X)_{hjt} \mathrm{B}} - \beta_1 \log e^{(0,X)_{hjt} \mathrm{B}} \right) \geq c^{A}_{jt} \quad \mathit{if}\ z_{jt} = 1$$

11. Return to step 5